Dynamic and Static Instabilities of Coherent Self-Oscillating System with Controlled Couplings[1]

S.S. Novikov

Tomsk State University, 36 Lenina av. Tomsk, 634050, Russia,
Phone: 7(3822)413964, Fax: 7(3822)423934, E-mail: lablia@npi.tpu.ru

Abstract - The results of task solution of local stability of synchronous states in the system of two strong coupled oscillators and conditions of elaboration of instabilities of different types are discussed in this report. The main feature of the model is the general description of wave coupling channel and resonance properties of the interaction parameter. It is demonstrated that the weakening of stability occurs in resonance coupling band, which can bring to dynamic instability. The conditions of occurrence of frequency (static) instability and competiting modes are determined.

1. Introduction

The problem of controlling relativistic devices microwave radiation parameters is nowadays relevant.
One of solutions is to introduce additional (external) coupling channel into resonance system [1-3]. The investigation of the system of interconnected oscillators allowed formulating principles, which postulate accordance between coupling channel generalized characteristics and existence of one or another forms (modes) of coherent oscillation [4,5]. In spite of certain universality of synchronous oscillations interaction power mechanism, such interactions have a wide range of variants in generating devices with advanced resonant system. External couplings appear as real mutual coupling channels, whereas internal couplings appear through the nonlinear mechanism of electron bunching. Nevertheless, if the system have resonance features and is regenerated from the direction of joint poles, it can be regarded as a system of interconnected oscillators. In the work [6] phenomenological construction of nonlinear current functions of that model is described. Linearization of these functions in all variables - amplitudes and phases of oscillation - enables to consider the problem of steady-state conditions local stability and to compare influence of external an internal couplings.

The most effective couplings with relation to stability are the resistance ones [7]; their realization supposes insertion of dissipative load-elements into coupling channels. However, real microwave coupling circuits are not wideband and have complicated frequency profile. It is shown in [4] that the interaction through elementary circuits has the clearly expressed resonant character and can bring to the loss of stability. It should be noted that this property of coherent systems is universal and is not dependent with coupling channel electrical length.

In this work, resonant features of the wave channel of mutual coupling with a dissipative non-uniformity (load) are analyzed and their influence on stability of coherent oscillation of two oscillators system is investigated in theory.

2. Two oscillators system

Two oscillators (Fig. 1) are coupled through circuit Y, which contains dissipative load-elements.

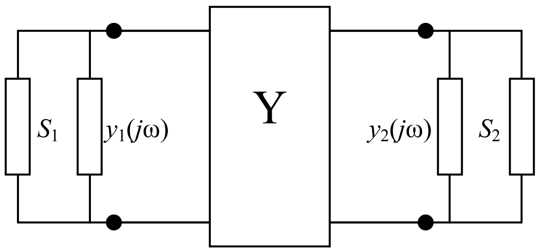

Fig. 1. Two oscillators system.

Complex admittance $y_k(j\omega)$ describes oscillators oscillating system features. It is supposed that due to their highly selective properties, the nearly harmonic process with the frequency ω_0 and slowly changing complex voltage amplitudes $\boldsymbol{U}_k = U_k(t)\exp(j\varphi_k(t)),\quad k=1,2$ is developed in the oscillator. Circuit Y is described by conductance matrix complex coefficient $y_{kk}(j\omega)$, $y_{kl}(j\omega)$. The abridged differential equation for $\boldsymbol{U}_k$ in symbolic form is the following:

$$\left[S_k(U_k) + Y_k(p) + Y_{kk}(p)\right]\boldsymbol{U}_k + Y_{kl}(p)\boldsymbol{U}_l = 0, \quad (1)$$

where $k \neq l$, $p = d/dt$, $S_k = -G_k + jB_k$ – are the conductivities of oscillator active elements averaged over the first harmonic. In accordance with the method of slowly changing amplitudes, the symbolic

[1] The work was supported by Russian Foundation for Basic Research (grant no. 05-08-01210a).

conductivity $Y_k(p)$ is obtained by the linear approximation of $y_k(j\omega)$ near ω_0:

$$Y_k(p) = y_k(j\omega_0) + 2C_k p,$$

where $2C_k = d(\operatorname{Im} y_k(j\omega))/d\omega\big|_{\omega_0}$. Parameter C_k is proportionate to oscillating system phase characteristic steepness and analogous to oscillatory circuit capacity.

Abridged operators $Y_{kk}(p)$ и $Y_{kl}(p)$ can be obtained from $y_{kk}(j\omega)$, $y_{kl}(j\omega)$ similarly.

3. Coupling channels resonance characteristics

It is evident that operators $Y_{kk}(p)$ and $Y_{kl}(p)$ cannot be regarded as independent; there functional binding should exist, as a result of the fact that the primary parameters $y_{kk}(j\omega)$, $y_{kl}(j\omega)$ are attributed to one circuit. Now let us introduce this bound in the general form.

Suppose that the interaction of oscillators is realized through wave channel with dissipative non-uniformity (load) (Fig. 2).

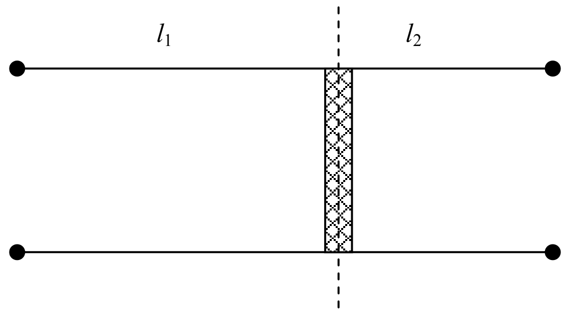

Fig. 2. Coupling channel.

In the event that the non-uniformity is thin the channel properties are defined by wave parameters of dispersion:

$$S_{11} = S'_{11}\exp(-2\gamma l_1),\ S_{22} = S'_{11}\exp(-2\gamma l_2),$$
$$S_{12} = S_{21} = S'_{12}\exp(-\gamma l_\Sigma), \qquad (2)$$

where $\gamma = \alpha + j\beta$, $\beta = 2\pi/\lambda$, λ is the wavelength in the channel; l_1, l_2, $l_1 + l_2 = l_\Sigma$ define the plane of load location and coupling channel length.

It is evident that coefficients S'_{kl} define the non-uniformity in the plane of its location; the following ratio is true for them:

$$S'_{11} = S'_{22},\ S'_{12} = S'_{21},\ S'_{11} - S'_{12} = -1,$$
$$S'_{11} + S'_{12} = r. \qquad (3)$$

Parameter r is non-uniformity reflectivity factor under in-phase of impinging waves.

Parameters $y_{kk}(j\omega)$, $y_{kl}(j\omega)$ with using (2) and (3) were draw:

$$y_{12} = -g_0\left[\frac{y_r}{g_0}\left(ch\gamma l_\Sigma - ch\gamma\delta l\right) + sh\gamma l_\Sigma\right]^{-1},$$
$$y_{11(22)} = -y_{12}\left[ch\gamma l_\Sigma + \frac{y_r}{g_0}\left(sh\gamma l_\Sigma \pm sh\gamma\delta l\right)\right], \qquad (4)$$

where $\delta l = l_2 - l_1$, $y_r = g_0(1-r)/(1+r)$ is equivalent dynamic conductivity of non-uniformity, g_0 – channel characteristic admittance; sign «–» corresponds with parameter y_{22}. Case $r = 0$ $(y_r = g_0)$ determines of common load (non-uniformity) matching and total addition of oscillator power within it; when $r < 0$ $(y_r > g_0)$ – the channel is overloaded; when $r > 0$ $(y_r < g_0)$ – the channel is underloaded. Fig. 3 shows dependencies of real and imaginary parts y_{12}/g_0 on channel electrical length $\theta_\Sigma = \beta l_\Sigma$ near $\theta_\Sigma = 2\pi$ for $r = 0, \delta l = 0$ estimated on (4). Evidently, coupling parameter y_{12} behaves in a resonance way. The width of resonant area $\operatorname{Re} Y_{12} < 0$ (it is the condition of stability of cophased and similar oscillations [4, 7]) for symmetrical system $(l_1 = l_2)$ contracts to a point if $\alpha \to 0$. Introduction of asymmetry $(l_1 \neq l_2)$ extends resonance area [4]. It should be marked that all parameters of quadripole possess one-order frequency properties.

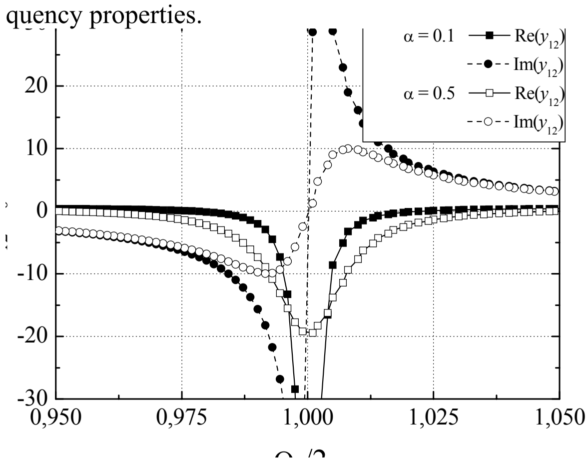

Fig. 3. Dependence of coupling parameter y_{12} on channel electrical length Θ_Σ.

Thus the symmetric coupling channel possesses substantial resonant properties, and at the same time channel electrical length needn't be large. The case $\alpha \to 0$ should be regarded as theoretical limits; calculation of linear losses is necessary for coupling channel description correctness.

In order to analyze equation (1) further lets write approximate expression (4) for symmetric ($l_1 = l_2 = l$) channel close to its «resonance frequency» ω_c (for it $\theta_\Sigma = 2\pi m, m = 1, 2, \ldots$, $\omega_c \approx \omega_0$):

$$y_{12} = -g_{12} + j2C_{12}\left(\omega - \omega_c\right),$$
$$y_{11} = y_r + g_{12} + j2C_{11}\left(\omega - \omega_c\right),$$
$$C_{11} = C_{in} - C_{12}, \qquad (5)$$
$$g_{12} = \frac{g_0}{\alpha 2l}, \quad 2C_{12} = \frac{g_0 l}{(\alpha l)^2 v},$$
$$2C_{in} = \left[1 - \left(\frac{y_r}{g_0}\right)^2\right]\frac{g_0 l}{v}.$$

Here v is speed of wave propagation.

Resonance properties of coupling channel are described in (5) by coefficients C_{12} and C_{11}. To estimate them lets write y_{12} in the form of

$$y_{12} = g_{12}\left(-1 + j2Q_c\frac{\omega - \omega_c}{\omega_c}\right),$$

where $Q_c = \omega_c C_{12}/g_{12} = 2\pi/\alpha\lambda_c$. It is clear that "Q-factor" of resonance is inversely proportional to losses for wave length. For microstrip line on dielectric spacers (the losses within of S-band is about $\alpha = 0,2 \div 0,5$) we have $Q_c \approx 300 \div 100$. When evidently the coupling channel on hollow waveguides ($\alpha < 0,01$) has very high Q_c. Therefore even if the coupling between oscillator and external channel is weak one shouldn't neglect coupling parameter resonance properties. And so C_{11} because of common load mismatching ($y_r \neq g_0$) may seem insignificant. Still we will take in into account, because, first, it won't complicate the problem, and, second, it can under certain conditions bring to instability of steady-state synchronous mode. It should be underlined also that the coupling value $y_{12} = -g_0/\alpha 2l$ (for $\omega = \omega_c$) according to the same estimation would greatly exceed typical system conductivity of g_0. Namely this property underlie of given in [4, 5] the definition of strong coupling.

4. Steady-state conditions under resonance couplings

The high sensitivity of the system for parameters deflection of oscillator with strong resonance coupling is result from the form of stationary equations. Lets write (1) for $p = 0$ by using (5) and assuming for simplicity sake that oscillators are phased in ($B_k = 0$), and coupling channel is matched $(y_r = g_0)$:

$$\left[-G_k\left(U_k\right) + g_0 + g_{12}\left(1 - \frac{\boldsymbol{U}_l}{\boldsymbol{U}_k}\right)\right] + $$
$$+j2\left[C_k\left(\omega_0 - \omega_k\right) - C_{12}\left(\omega_0 - \omega_c\right)\left(1 - \frac{\boldsymbol{U}_l}{\boldsymbol{U}_k}\right)\right] = 0.$$

If case of ideal tuning ($\omega_c = \omega_1 = \omega_2$) equation have cophased solution ($\varphi_2 - \varphi_1 = 0$) even for $U_2/U_1 \neq 1$; at the same time the greater g_{12} the closer the amplitudes will approach. But if coupling circuit is detuned ($\omega_c \neq \omega_1 = \omega_2$), synchronous frequency ω_0 will be unequal to oscillator frequencies. This distinction can be substantial in case of major resonance properties, i.e. $C_{12} > C_k$. In this situation even a small frequency detuning of oscillators ($\omega_2 \neq \omega_1$) will sharply change of equation symmetry. Then the misphasing of oscillation ($\varphi_2 - \varphi_1 \neq 0$) is changing amplitude ratio and bring to greater detuning from resonances. As a result of that the systems can demonstrate complicated behavior even within a narrow band of mutual frequency detuning. The typical features of strong coupling systems are anomalously sharp relation of synchronous frequency during oscillators parameter or coupling circuit disturbances, and also disruption of synchronism by means of oscillations suppression. The concerned instability is conditioned by high sensibility of stationary synchronous modes of the system to deflections of its parameters. It can be stated that strong resonance coupling makes the system less coarse.

5. Dynamic and static instabilities

An important element of investigating dynamic systems is instability analyze. When applying to (1) a standard procedure of linearization and substituting solution $\delta a_k^* \exp(\lambda t)$, we have a system of algebraic equations for deflection of amplitude δa^* and phases $\delta\varphi^*$ and a characteristic equation for parameter λ. The fullest information about system local features can be taken from immediate determination of roots for characteristic equation and corresponding vector-solutions. They define the direction of deflaction in phase space, where one or another system's couples are being tested. Analytical solution of the problem is possible for cophased $(\varphi_2 - \varphi_1 = 0)$, equiamplitide $(U_2/U_1 = 1)$ stationary oscillations, when: system is symmetric $(l_1 = l_2)$, oscillators are identical $(G_1 = G_2 = G, C_1 = C_2 = C)$ and are phased $(B_k = 0)$, and coupling channel is tuned $(\omega_0 = \omega_1 = \omega_2 = \omega_c)$. The roots are the following:

$$\lambda_1 = 0,$$
$$\lambda_2 = \sigma/2\left(C + C_{in}\right),$$
$$\lambda_3 = -g/\left(C - 2C_{12}\right),$$
$$\lambda_4 = \left(\sigma - 2g\right)/2\left(C - 2C_{12}\right);$$

where $\sigma = U_0\, dG/dU \big|_{U_0} < 0$ is parameter, referred to as limiting cycle strength.

The first root $\lambda_1 = 0$ corresponds to monodirectional phases disturbance ($\delta a_1^* = \delta a_2^* = 0$, $\delta\varphi_1^* = \delta\varphi_2^* \neq 0$), toward which the system is neutral. Equal disturbance of stationary amplitudes ($\delta a_1^* = \delta a_2^* \neq 0$, $\delta\varphi_1^* = \delta\varphi_2^* = 0$) corresponds to root λ_2. The constraint reaction here is excluded therefore parameter C_{12} doesn't belong to λ_2. Coupling channel frequency features in these variations may appear only on its mismatch, i.e. when $C_{in} \neq 0$. In reality, if the channel is matched ($y_r = g_0$, see (5)) and $C_{in} = 0$, we have a well-known condition of amplitude stability of oscillator $\sigma < 0$. The stability in this direction will be lost ($\lambda_2 < 0$), if the channel is overloaded ($C_{in} < 0$) and $|C_{in}| > C$. It is important to mark that value C_{in} might be great, even for the small mismatching, if the channel length long enough. The type of root λ_2 shows that the loss of stability occur on changing of phase-frequency characteristic's sign in the oscillation system resonance area. Given instability has hysteresis character with the elements of frequency pulling. Apparently, that is why it is called frequency or static. In the same time as root λ_2 describes system reaction on amplitude disturbance, the discussed inequality has the meaning of amplitude condition.

Root λ_3 describes system reaction on reverse disturbance of phases ($\delta a_1^* = \delta a_2^* = 0$, $\delta\varphi_1^* = -\delta\varphi_2^* \neq 0$), therefore frequency parameter C_{12} of coupling conductivity $y_{12}(j\omega)$ belongs to λ_3. Evaluations given above show that cophased coherent oscillations in symmetric system with strong coupling might be stable only when the long attenuation in channel are substantial. The loss of stability in this direction comes from coherence destruction and can be referred as dynamic instability. The type of expression for root λ_3 gives explanation to this instability. The positive sign of parameter C_{12} (see (5)) corresponds with series resonance. Autooscillations on this type of resonance (if it prevails: $2C_{12} > C$) may exist only for active elements, which have S-shaped current-voltage characteristics, i.e. $\sigma > 0$.

Root λ_4 characterizes local motion in reverse disturbance of amplitudes ($\delta a_1^* = -\delta a_2^* \neq 0$, $\delta\varphi_1^* = \delta\varphi_2^* = 0$). Resonance features of coupling parameter dominate here too. Summarizing the analyze we should note that if coupling circuit frequency characteristics will be formally neglected ($C_{12}, C_{in} < C$), then necessary condition of cophased oscillations stability [7] come from the obtained solutions:

$$\operatorname{Re} y_{12}(j\omega) = -g_{12} < 0 .$$

6. Conclusion

In this work, local stability of synchronous oscillations of two strongly coupled oscillators and conditions of the development of different instabilities have been analyzed. It has been shown that the generalized symmetric wave-coupling channel with a common load has resonant features, which are commensurable or exceed ones for oscillators' oscillation systems. In this case, coherent regime is impossible since the oscillation phase difference is unstable (dynamic instability). Mismatch of common load under certain conditions may lead to the hysteretic (static) instability. High sensitivity of amplitude-phase stationary characteristics of strongly-coupled systems to the variation of their active and passive elements is noticed.

References

[1].I.I. Vintizenko, A.I. Zarevich, S.S. Novikov, *Patent RU N 2190281,* 2002.

[2].I.I.Vintizenko, V.I. Guselnikov, A.I. Zarevich, S.S. Novikov, Letters to JTF, **29**/7, 64 (2003).

[3].I.I. Vintizenko, A.I. Zarevich, S.S. Novikov, *in Proc. 13th Symposium on High Current Electronics*, Tomsk, Russia, 2004. pp. 300-303.

[4].S.N. Vladimirov, A.S. Maidanovskii, S.S. Novikov, *The nonlinear oscillations of multifrequency autooscillators*, Tomsk, Tomsk University Publishing House, 1993, 203 p.

[5].S. A. Maidanovskii, S. S. Novikov, Radiotechnika I electronica, **48**/5, 1 (2003). pp. 595-600.

[6].Zarevich A.I., Vintizenko I.I., Novikov S.S., *in Proc. 13th Symposium on High Current Electronics*, Tomsk, Russia, 2004. pp. 273-276.

[7].A.A. Dvornikov, G.M. Utkin, *Phased self-oscillator of radio transmitting equipment*, M. Energy, 1980, 177 p.