Novel Scheme of Bragg FEM Based on Coupling of Propagating and Trapped Waves[1]

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In this paper we discuss a new variant of FEM based on the interaction between a propagating wave and a quasi cut-off trapped wave. This coupling is realized by either helical or azimuthally symmetrical corrugation. The quasi cutoff mode provides the feedback leading to the self-excitation of the whole system while the efficiency in a steady-state regime of generation is almost completely determined by the propagating mode, synchronous to the beam. Analysis based on averaged time domain approach as well as on direct PIC code simulation shows that the efficiency of such a device in the single mode single frequency regime can be rather high. The main advantage of the novel Bragg resonator is provision of higher selectivity over transverse index than traditional scheme of Bragg FEM. "Cold" microwave tests of the Bragg structure based on coupling of propagating and trapped waves in the Ka-band demonstrates good agreement with theoretical consideration.

1 Introduction

A number of oscillator schemes based on the interaction between propagating and a quasi cutoff trapped waves are known in microwave electronics. One of them is a scheme of a gyrotron [1] where an electron beam excites a quasi cutoff mode while the output of radiation is provided by the propagating wave coupled with the trapped one via corrugation of the waveguide side walls. One more example is a scheme of cyclotron resonance maser (CRM) [2] and FEM [3] in which the electron beam interacts both with a propagating wave (at the first harmonic) and a cutoff mode wave (at the second or third harmonic). In this case direct coupling of the electromagnetic waves is absent and the waves interact via electron beam modulation. The trapped cutoff mode provides an oscillator self-excitation while the propagating wave is responsible for energy extraction in a steady-state regime.

In this paper we discuss a new variant of FEM based on the interaction between a propagating wave and a quasi cutoff trapped wave [4]. In a suggested scheme a beam of wiggling electrons interacts only with a propagating wave, but the latter is coupled to a quasi cutoff mode. This coupling could be realised by either helical or azimuthally symmetric periodical waveguide corrugation. The quasi cutoff mode provides the feedback mechanism leading to the self-excitation of the whole system while the efficiency in steady-state regime of generation is almost completely determined by the interaction with the propagating wave, synchronous to the beam.

The main advantage of the suggested scheme is provision of higher selectivity over transverse index than traditional scheme of FEM with Bragg resonators where a feedback wave propagates in backward direction with rather high group velocity [5]. At the same time, this scheme is able to provide higher Doppler up-shift in comparison with the scheme discussed in [2, 3], where the frequency is restricted by a number of the operating harmonic. The novel feedback scheme will be tested at a JINR- IAP FEM [6, 7] as a method of increasing oscillation frequency for fixed transverse size of interaction space.

Both helical and axial-symmetric corrugations can be used to realize coupling of propagating and trapped waves. Nevertheless in the case of helical corrugation the trapped wave scatters only in forward propagating wave. In the case of axial-symmetric corrugation trapped mode scatters both in forward and backward propagating waves. As a result such corrugation can be used to realize narrow band reflector for a traditional two-mirrors FEM scheme. In the helical case the RF oscillations should be excited by electron beam directly inside corrugation section. This process is described in the Sect.2. Sect.3 is devoted to the KARAT PIC simulation of a planar geometry FEM in the corrugated waveguide. In Sect.4 the simple analytical model of a Bragg reflector exploiting coupling of propagating and cutoff mode is developed and compared with results of experimental testing.

2 FEM oscillator based on coupling of propagating and trapped waves on helical corrugation

The fields of a propagating (index A) and a trapped quasi cutoff mode (index B) can be presented as

$$\vec{E} = \mathrm{Re}\left(A(t,z)\vec{E}_{\mathrm{A}}(r_\perp)\mathrm{e}^{-ihz-im_{\mathrm{A}}\varphi}\mathrm{e}^{i\omega t}\right) \quad (1)$$

$$\vec{E} = \mathrm{Re}\left(B(z,t)\vec{E}_{\mathrm{B}}(r_\perp)\mathrm{e}^{-im_{\mathrm{B}}\varphi}\mathrm{e}^{i\omega t}\right), \quad (2)$$

where ω is the carrier frequency, which is chosen equal to the eigenfrequency of the trapped mode and

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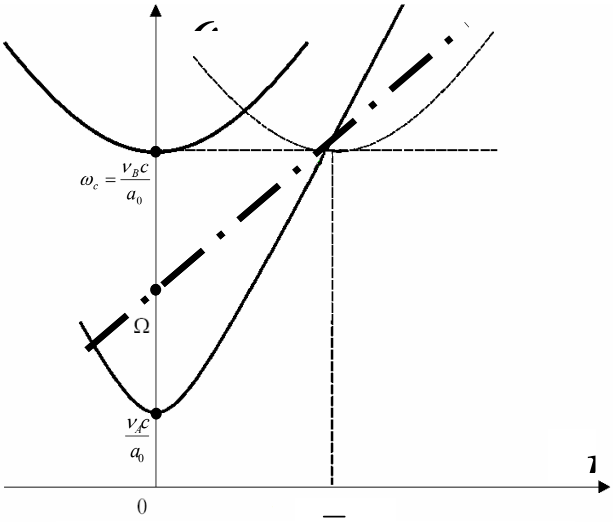

Fig.1. Dispersion diagram showing coupling of waves

$\vec{E}_{A,B}(r_\perp)$ functions specify the transverse structure of the modes.

Helical corrugation $r = r_0 + r_1 \cos(\bar{h}z + \bar{m}\varphi)$ (r_0 is the mean radius of the waveguide, $\bar{h} = 2\pi/d$, d and $2r_1$ are the period and the depth of the corrugation correspondingly) under the Bragg resonance conditions

$$h \approx \bar{h}\,,\ \ \bar{m} = m_1 - m_2 \qquad (3)$$

provides the coupling of the waves (1) and (2) as shown on dispersion diagram in Fig.1.

Forward propagating wave is synchronous to the electrons moving in the $+z$ direction: $\omega_0 - h_0 v_\parallel \approx \Omega_b$, where Ω_b is the frequency of particle oscillations in a spatially periodic undulator field for FEL (FEM) or in homogenous magnetic field for CRM.

The process of amplification of synchronous wave and the mutual coupling of the propagating (2) and trapped (1) modes can be described by the system of equations in which the evolution of amplitude of the cutoff mode *B* is described by the parabolic equation :

$$\frac{\partial \hat{A}}{\partial Z} + i\delta\hat{A} = i\alpha\hat{B} + C^3 J \qquad (4a)$$

$$\frac{\partial \hat{B}}{\partial \tau} + \frac{i}{2}\frac{\partial^2 \hat{B}}{\partial Z^2} + \sigma\hat{B} = -i\alpha\hat{A}(Z)\ , \qquad (4b)$$

In oscillator the amplitudes of the propagating wave is zero at the edge of interaction space: $\hat{A}\big|_{Z=0} = 0$. Boundary conditions describing the cutoff mode in the waveguide without the cutoff narrowings are

$$B + \frac{1}{\sqrt{\pi i}}\int_0^\tau \frac{\partial B(\tau')}{\partial Z}\frac{d\tau'}{\sqrt{(\tau-\tau')}}\Bigg|_{Z=0,Z=L} = 0\,. \ (5)$$

Stationary modes of oscillations can be found from the system (4) with these boundary conditions. Spatial structures of these modes and efficiency vs. detuning are presented in Fig. 2.

Initial condition can be defined as a small "seed" field of the trapped mode: $\hat{B}\big|_{\tau=0} = \hat{B}_0(Z)$. A dimensionless amplitude of RF current, $J = \frac{1}{2\pi}\int_0^{2\pi} e^{-i\theta(Z)}d\theta_0$ is determined by the electron motion equations:

$$\frac{\partial^2\theta}{\partial Z^2} = \mathrm{Re}\left(\hat{A}_+(Z)e^{i\theta}\right).$$

$$\theta\big|_{Z=0} = \theta_0 \in [0,2\pi),\ \frac{\partial\theta}{\partial Z}\bigg|_{Z=0} = \Delta \qquad (5)$$

where Δ is the initial mismatch of the electron-wave synchronism. In Eqs. (4-6): $Z = z\bar{h}$, $\tau = \bar{\omega}t$, $\hat{A} = e\kappa\mu A / mc\bar{\omega}\gamma_0$, $\hat{B} = e\kappa\mu B\sqrt{N_B} / mc\gamma_0\sqrt{N_A}$, $\theta = \omega_0 t - h_0 z - \int \Omega_b \mathrm{d}t$ is the electron phase in the field of the synchronous wave, $C = \left(\frac{eI_0}{mc^3}\frac{c\lambda^2 K^2\mu}{4\pi^2\gamma_0 N_A}\right)^{1/3}$ is the Pierce parameter, I_0 is the unperturbed beam current, μ is the parameter of inertial bunching, κ is the wave-electron coupling parameter proportional to the amplitude of the electron transverse oscillations, α - is the wave coupling coefficient at the Bragg grating proportional to its depth r_1, $N_{A,B}$ are the norms of A and B modes respectively, $\delta = (\bar{\omega} - \omega_c)/\bar{\omega}$ is the normalized mismatch of the Bragg frequency, $\bar{\omega} = \bar{h}c$, σ is a coefficient of the Ohmic losses for the cutoff mode.

In eqs (4-5) we assumed that time Q/ω of the changing of the trapped mode amplitude B to be substantially greater than the electrons transit time $l/v_\parallel$ and forward wave propagation time l/v_{gr}. In this case it is possible to neglect time derivatives in the equations of electron motion (6) as well as the forward wave excitation equation (4a). Stationary oscillations regime can be found from the system (4) (6) with boundary conditions (5). Efficiency in stationary oscillations regime vs. electron detuning are presented in Fig.2a. Spatial structures of partial waves for Δ=-2 is shown in Fig 2b. It is important to note that in optimal conditions the main part of the energy extracted from electron beam transforms in the radiation of the propagating mode *A* but not dissipated with the trapped mode *B*.

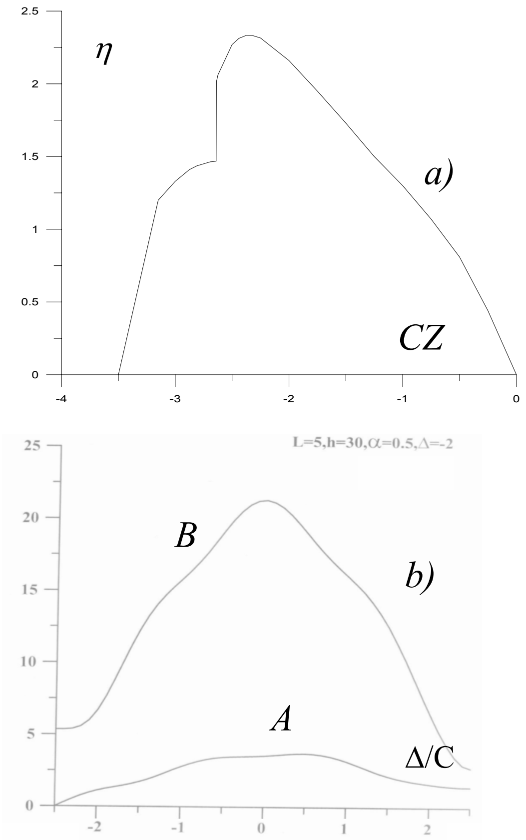

Fig.2.The efficiency in stationary regime vs. electron detuning (a) and the mode structure at *L*=5, *C*=0.03 , α=0.5, Δ=-2 (b).

Using above analysis let us estimate a possibility of creation of an 4mm FEM on the basis of the accelerator LIU-3000 (JINR, Dubna) (1 MeV / 200 A / 200 ns). Taking waveguide radius $r_0 = 0.65$ cm, the period of undulator $d_u = 3$ cm and the period of helical corrugation $\overline{m} = -2$, $d \approx 4$ cm we find that the conditions of a Bragg resonance (4) are fulfilled for the pair of modes $TE_{11} \to TM_{13}$. In the case of Pierce parameter $C \approx 10^{-2}$ the length of interaction space $l = 20$ cm and corrugation depth $a_1 \approx 0.01$ cm corresponds to the normalized length $CL = 2.5$ and the coupling parameter $\alpha = 0.001$. The relative mismatch of the Bragg frequency from the cutoff frequency $\delta = (\overline{\omega} - \omega_0)/\omega_0 \approx -0.01$. Total efficiency in this regime amounts $\eta \approx 15\%$.

3 KARAT PIC simulation of a planar FEM model

An FEM with coupling of a propagating and a quasicritical modes was simulated using a KARAT PIC code. We used a planar 2D model, in which the lowest TEM mode was coupled to the H2 mode of a planar waveguide via the 8 *mm*-periodic corrugation of the waveguide walls. Boundaries of the waveguide were matched (i.e. without cutoff narrowings). Excitation of the second longitudinal mode was observed at the parameters *l=40 cm, f=37GHz,* which are close to those of the planned experiment. The efficiency in the steady-state regime reached 20%. Results of the simulation is presented in Fig.3.

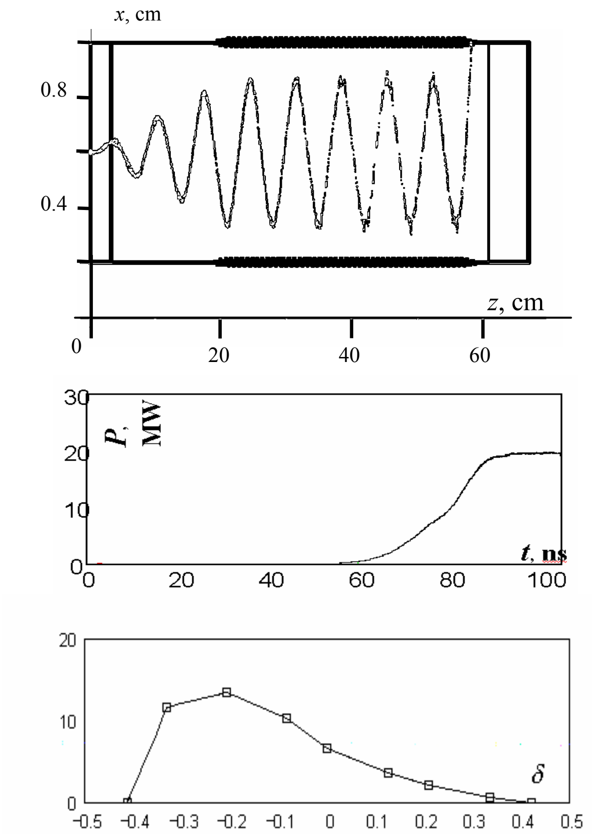

Fig.3. PIC simulation: the system geometry (up), establishment of a steady-state regime (middle) and the dependence of the efficiency on δ (down)

4 Narrow band reflector based on coupling of propagating and cutoff mode on symmetric corrugation

In the case of symmetric corrugation $\overline{m} = 0$ the coupling of propagating and cutoff modes can be used to realize a narrow band reflector. In this case cutoff mode scatters both in forward and backward propagating waves:

$$\vec{E} = \mathrm{Re}(A_+(t,z)\vec{E}_A(r_\perp)\mathrm{e}^{i\omega t - ihz} + A_-(t,z)\vec{E}_A(r_\perp)\mathrm{e}^{i\omega t + ihz})$$

In the case of monochromatic signal this process is described by the equations (compare to Eqs. (4)):

$$\frac{dA_\pm}{dZ} \mp i\Omega A_\pm = \pm i\alpha B e^{\pm i\delta Z}$$

$$\frac{1}{2}\frac{d^2B}{dZ^2} + \Omega A_\pm = \alpha(A_+e^{-i\delta Z} + A_-e^{-i\delta Z}) \quad (7)$$

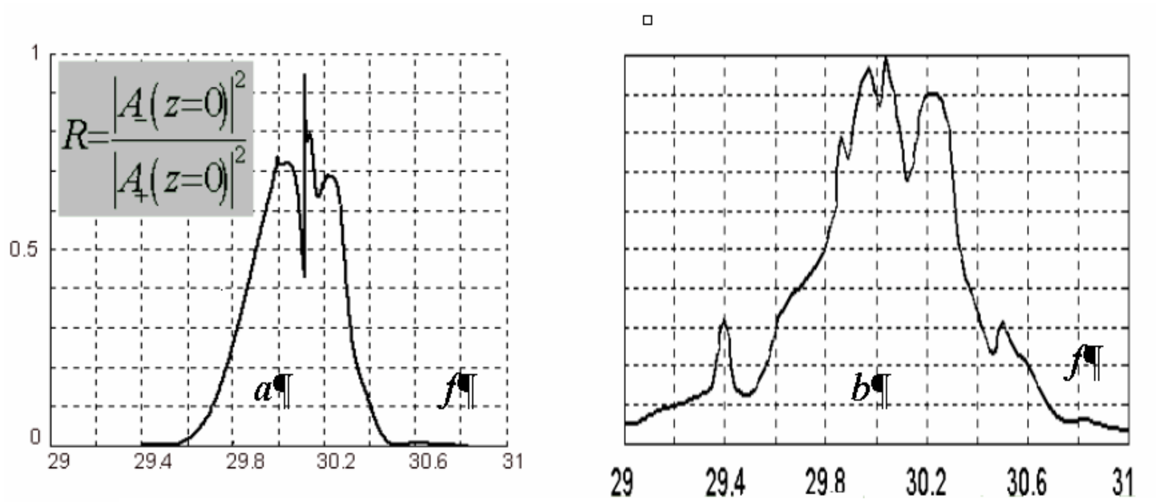

Fig. 4. Frequency dependence of the reflection coefficient: (a) results of simulation, (b) results of 'cold' testing.

with boundary conditions:

$$A_+\big|_{Z=0} = A_0 \qquad A_-\big|_{Z=L} = A_0 ,$$

$$\left.\frac{dB}{dZ} \mp i\sqrt{2\Omega}B\right|_{Z=0,Z=L} = 0 \cdot$$

Here Ω is normalized frequency detuning.

Equations (7) were simulated at the parameters of a Bragg structure with the length *l=20 см*, corrugation depth was *0.3 mm*, period was *10.7 mm,* which makes the calculated coupling coefficient between the H_{11} and H_{12} $\alpha \approx 6{,}4\times10^{-3}$. These parameters are close to the ones used in the experimental testing. Fig. 4a shows frequency dependence of the reflection coefficient. Comparison of simulation results with results of the experimental K band measurements shown in Fig. 4b. Thus, using shallow symmetric corrugation and coupling propagating and cutoff mode it is the possible to realize of the narrow-band reflector with the reflection band of 400 MHz 3D code simulation confirms this fact and indicates that the reradiaton to the parasitic waveguide modes does not exceed 5% power.

Conclusion

As follows from above consideration, a shallow corrugation ($r_1/r_0 \approx 0.03$) is sufficient for FEM self-excitation if the trapped cutoff mode is used to form a feedback loop. Thus the using of a cutoff mode makes it possible to decrease effective coupling parameter sufficiently enough for the self-excitation of the oscillator in comparison with traditional Bragg FEM scheme, where feedback wave possesses rather high group velocity [5]. Correspondingly in oversized microwave system where Bragg conditions are satisfied for a large number of pairs of waves with different transverse indices it is possible to provide selective excitation of a single pair consisting of a cutoff mode and operating propagating mode which is amplified by the electron beam. Above method of mode control will be tested in JINR-IAP FEM [6, 7] at Ka-band and then used as a method of increasing the operating frequency to W-band for fixed transverse size.

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