Cherenkov Interaction of a Hollow Relativistic Electron Beam with RF Fields of a Disc-Loaded Rod Antenna in a Circular Waveguide[1]

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Abstract – The instability of an electron beam propagating within the slow-wave supporting structure comprising an inner disc-loaded conducting rod and an outer circular waveguide is considered. The linear theory is developed in the one-wave approximation, i.e., not taking the space harmonics into account. The complex solutions of the derived dispersion relation are investigated numerically for the fundamental mode of the system, the quasi-TEM mode. The results are compared with the case of an inner dielectric rod, for which the fundamental mode is the non-axisymmetric HE_{11} mode. The possibility of using the disc-loaded rod antenna in a hybrid antenna-amplifier device is discussed. For the disc-loaded conducting rod, spatial growth rates are higher due to stronger beam-wave coupling at given beam-rod distances, and amplification bands are wider due to weaker dispersion of the quasi-TEM mode; however, for the dielectric rod, the issues of the microwave drive signal input are significantly easier.

1. Introduction

It has been recently proposed that a surface wave antenna could be combined with a traveling wave tube if a relativistic electron beam propagates near and along the antenna's slow-wave structure. In such a hybrid antenna-amplifier device, the antenna feed signal would simultaneously serve as the traveling wave tube RF drive. To explore this concept, the linear theory was developed earlier [1] for the case of a dielectric rod antenna with its fundamental non-axisymmetric HE_{11} operating mode. Meanwhile, since a dielectric structure can suffer from surface charge build-up when interacting with an electron beam, it is of interest to consider the case of a conducting surface wave antenna and compare achievable values of gain and bandwidth.

In this work, the linear theory is developed for the case of a disc-loaded rod antenna in a circular waveguide. Like a dielectric rod antenna, the disc-loaded conducting rod represents the well-developed kind of surface wave antennas [2]. The important difference is in the fact that for the system with the dielectric rod, the lowest, dipole HE_{11} mode is very well separated from other modes in frequency corresponding to a given phase velocity [1]. The system with the conducting rod, as a coaxial system, supports propagation of the axisymmetric quasi-TEM mode, which has no cut-off frequency like the TEM-mode of the coaxial, but has an axial electric field component and a slight dispersion of slowed down phase velocity. This mode will be fundamental in the Cherenkov interaction with the electron beam; however, the non-axisymmetric HE_{11} mode of this system may have close frequencies of velocity synchronism that may lead to a strong mode competition.

The consideration below is carried out using the simplified model of infinitely small period of the system, i.e., not taking the space harmonics into account (one-wave approximation). Under this assumption, non-axisymmetric modes having all six RF field components cannot be considered because for them, the axial magnetic field cannot be constant between the discs due to the boundary conditions at the disc surface. Therefore the initial consideration is limited by the case of azimuthal symmetry.

It is interesting to note that the structure under consideration here was used in the very first, pioneer experiment [3] on high-power microwave generation. Later, the system with a corrugated rod was successfully used [4] for the generation in the short millimeter wavelength range.

2. Dispersion Relation

The geometry of the system is depicted in Fig. 1. The hollow electron beam is assumed to be infinitely thin, fully magnetized, and monoenergetic.

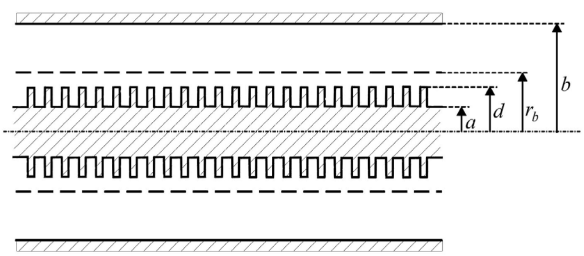

Fig. 1. Axial cross-section of the structure. The thin annular electron beam is shown by the dashed line

[1] The work was supported by the European Office of Aerospace Research and Development (EOARD) through the ISTC Partner Project No. 2823p.

In the one-wave approximation, the RF fields in the region between two neighbor discs do not depend on the axial coordinate. Taking this into account, one can find from the wave equation the following solutions for the axial electric field component as a function of the radial coordinate r:

$$E_z(r) = \begin{cases} E_0 \dfrac{F(ka,kr)}{F(ka,kd)}, & a \le r \le d; \\ E_0 \dfrac{\Phi(qr,qr_b)}{\Phi(qd,qr_b)} + E_b \dfrac{\Phi(qd,qr)}{\Phi(qd,qr_b)}, & d \le r \le r_b; \\ E_b \dfrac{\Phi(qr,qb)}{\Phi(qr_b,qb)}, & r_b \le r \le b. \end{cases}$$

Here $k = \omega/c$, $q^2 = h^2 - k^2$, ω and h are the circular frequency and axial wavenumber, c is the speed of light, E_0 and E_b are the field values at the radii of the disc edges and beam, respectively, and functions F and Φ are defined as:

$$F(x,y) = J_0(x)Y_0(y) - J_0(y)Y_0(x),$$

$$\Phi(x,y) = I_0(x)K_0(y) - I_0(y)K_0(x),$$

where J_0, Y_0 are Bessel functions, and I_0, K_0 are modified Bessel functions of zero order.

These solutions are matched using the condition of the continuity of $H\varphi$ field component at the radius of the disc edges and the condition for the derivative jump at $r = r_b$ obtained from the linearized fluid equations for the beam [1]:

$$\left\{\frac{dE_z}{dr}\right\}_{r=r_b} = -\frac{2I_b}{\beta\gamma^3 I_A}\frac{q^2}{(k-\beta h)^2}\frac{E_b}{r_b},$$

where I_b is the beam current, $I_A = mc^3/e$, m and e are the electron rest mass and charge, β is the normalized beam velocity, and γ is the Lorentz factor.

Simple but tedious formula manipulations finally yield the dispersion relation of the system:

$$\left[\frac{k}{q}\frac{\Phi'(qb,qd)}{\Phi(qd,qb)} - \frac{F'(ka,kd)}{F(ka,kd)}\right]\frac{(k-\beta h)^2}{\Phi(qr_b,qb)} =$$

$$= \frac{2q^2 I_b}{\beta\gamma^3 I_A}\frac{\Phi(qd,qr_b)}{\Phi(qd,qb)}\left[\frac{F'(ka,kd)}{F(ka,kd)} - \frac{k}{q}\frac{\Phi'(qr_b,qd)}{\Phi(qd,qr_b)}\right],$$

where $F'(x,y) = J_0(x)Y_0'(y) - J_0'(y)Y_0(x)$,

$$\Phi'(x,y) = I_0(x)K_0'(y) - I_0'(y)K_0(x).$$

The results of the numerical investigations of this dispersion relation are presented in the next section.

3. Numerical Results

Achievable values of gain and bandwidth in the quasi-TEM operating mode are well illustrated in comparison with those achievable in the structure with the dielectric rod in its lowest HE_{11} mode. In Figs. 2 and 3, the results of growth rate calculations are shown obtained for the same electron beam and the same radii of disc edges (for the disc-loaded rod) and dielectric surface (for the dielectric rod); the depth of grooves in the disc-loaded rod is varied. In Fig. 2, the parameters of the beam ($\gamma = 1.4$, $I_b \approx 1$ kA) and rod ($d/b = 0.3$) correspond to the case of the HE_{11} mode domination in the instability spectrum presented in [1] for the system with the dielectric. For the curve 1 in Fig. 2, the depth of grooves is such that the peak gain frequencies in the cases of disc-loaded rod and dielectric rod (dashed curve) are very close. It is seen that for the disc-loaded rod, the gain is much higher and the amplification band is significantly wider than for the dielectric rod at all other parameters fixed. As the depth of grooves reduces (curve 2), the amplification band shifts to higher frequency, the relative bandwidth becomes narrower, and the peak gain decreases.

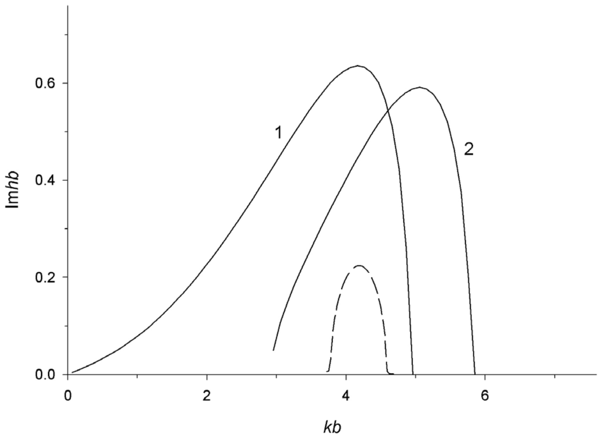

Fig. 2. Spatial growth rate versus frequency for the thin hollow beam propagating between the disc-loaded rod and circular waveguide (solid curves). The parameters are: $\gamma = 1.4$, $I_b/I_A = 0.06$, $r_b/b = 0.5$, $d/b = 0.3$, $a/b = 0.15$ (1), 0.17 (2). The dashed curve shows the growth rate of the HE_{11} mode of the system with the dielectric rod having the same radius $d/b = 0.3$ and the permittivity $\varepsilon = 5$ calculated at the same beam parameters

The fact that much higher gains are achievable for the disc-loaded rod is naturally explained by stronger beam coupling to the synchronous mode at a given distance between the beam and rod surface. Indeed, the quasi-TEM mode is the mode of pure TM-type, whereas the HE_{11} mode of the structure with the dielectric rod is the hybrid mode originated from the TE_{11} mode of a circular waveguide, which has no axial electric field component; hence, the coupling impedance of the HE_{11} mode cannot be as high as that of the quasi-TEM mode. The wider bandwidth achievable with the disc-loaded rod can be explained by a significantly weaker dispersion of the quasi-TEM mode in comparison with the HE_{11} mode of the system with the dielectric rod. Indeed, the quasi-TEM mode has no cut-off frequency, and its phase velocity is always subluminal that is not the case with the HE_{11} mode. Moreover, for the dashed curve of Fig. 2, the permittivity is rather large ($\varepsilon = 5$) that contributes into stronger dispersion. In Fig. 3, the comparison is carried out for the case of smaller permittivity of the dielectric rod, and it is seen that the bandwidths corresponding to the dashed and solid curves are of the same order. However, the peak gain remains higher for the case of disc-loaded rod in the same degree as for the parameters of Fig. 2. In addition, for the dielectric rod, the parameters correspond to the case of no prevalence of the HE_{11} mode over higher-order modes in the instability spectrum [1].

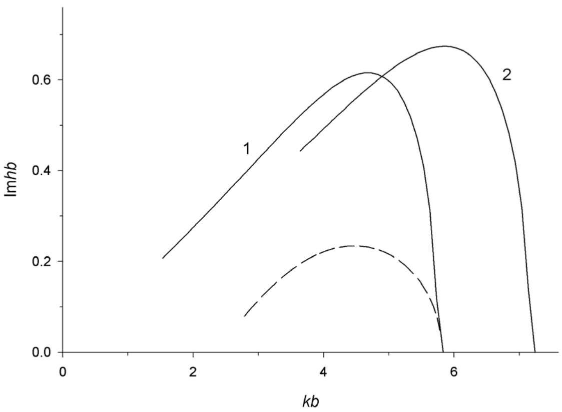

Fig. 3. The same as in Fig. 2 except for the parameters: $\gamma = 1.8$, $I_b/I_A = 0.1$, $r_b/b = 0.6$, $d/b = 0.5$, $a/b = 0.375$ (1), 0.4 (2). For the dielectric rod (dashed curve), the radius $d/b = 0.5$ and the permittivity $\varepsilon = 2.25$

It should be noted that, in difference of Fig. 2, reducing the depth of grooves at the parameters of Fig. 3 does not result in decreasing peak gain, but on the contrary, the peak gain increases. This is due to higher beam energy and shorter beam-rod gap keeping the beam within the scale of evanescence of the surface wave even at the reduced wavelength corresponding to the velocity synchronism.

4. Discussion

As is seen from the comparison presented above, the achievable gain in the quasi-TEM mode of the structure comprising the disc-loaded rod antenna and outer circular waveguide is much higher than in the HE11 mode of the waveguide with the dielectric rod. At the waveguide dimension giving the X-band operating frequency, the gain per unit length reaches ~ 3 dB/cm. In addition, the bandwidth for the disc-loaded rod can be considerably wider under certain parameters. That means one could employ the conducting surface wave antenna to realize the concept of hybrid antenna-amplifier device.

Nevertheless, there are significant difficulties connected with the problem of the microwave drive signal input in the case of conducting rod antenna. The key feature of the antenna-amplifier concept making the device compact [5] is that a rod antenna feed waveguide serves, at the same time, as a hollow cathode holder of a diode producing a relativistic electron beam under applied voltage generated using a module of a linear induction accelerator. Unlike the dielectric rod, the conducting rod should not be in contact with the cathode; otherwise the beam is not able to drift through the interaction space. If the rod is distanced from the cathode (i.e., the horn of antenna feed waveguide), it would result in reflection losses of the RF drive signal. One could use some insulating supports for the rod inside the cathode holder; in this case, there is a problem of possible emission from its inner surface.

Another factor contributing to the problem of RF drive input in the case of disc-loaded rod antenna is the excitation of the non-axisymmetric HE_{11} mode mentioned above in the introduction. To avoid a multi-mode transmission of the feed signal, it is more natural if it propagates in the TE_{11} mode of the circular feed waveguide. However, if the HE_{11} mode is operating, the competition of the quasi-TEM mode can be very significant.

In spite of these difficulties, a solution can be found allowing for the use of the conducting rod antenna in the antenna-amplifier device. Though, it requires additional investigations. It is of particular importance to develop the linear theory similar to presented here, but taking into account the space harmonics to consider the Cherenkov interaction of the beam with non-axysimmetric modes of the structure with the disc-loaded rod.

References

[1] A.S. Shlapakovski, Tech. Phys. Lett. **25**, 267 (1999) [Pis'ma v ZhTF **25**(7), 43 (1999)].

[2] Carlton H. Walter, *Traveling Wave Antennas*, Moscow, Energiya, 1970 (*translation into Russian*).

[3] J.A. Nation, Appl. Phys. Lett. **17**, 491 (1970).

[4] N.I. Gunina, S.D. Korovin, S.D. Polevin et al., Pis'ma v ZhTF **14**, 1425 (1988).

[5] A.S. Shlapakovski, I.I. Vintizenko, A.V. Petrov, E. Schamiloglu, *in: Proc. 13th Symp. on High-Current Electronics*, 2004 (this issue).