Local Oscillation Stability in Magnetron with Coupled Cavities

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Abstract – In this report a self-oscillatory model of a magnetron with coupled cavities is presented. In magnetron's resonance system two oscillatory subsystems are distinguished that interact with each other through internal and external circuits. Phenomenological description of subsystems' synchronous interaction is given. The influence of coupling on local stability of coherent oscillations is researched.

1. Introduction

Resonance systems of many electrovacuum generating microwave devices of microwave band are complex electrodynamic systems, which are characterized with eigen oscillation types referred to as modes. These modes differ by field distribution in the resonance system, and their frequency spectrum can be quite dense. This causes instability of microwave radiation, as well as of transitions between competing oscillation modes, frequency jumps and power surges. This problem is especially typical for pulse devices, including high power relativistic generators [1, 2]. There are methods of modes separation, aimed at oscillating systems' modification with the purpose of diluting the eigenfrequency spectrum [3, 4].

The subject of this report is a self-oscillatory model of a magnetron with coupled cavities. The influence of coupling on local stability of coherent oscillations is researched.

2. Self-Oscillatory Model of a Two-Output Magnetron

In certain cases, the mechanism of energy transmission into electromagnetic oscillations allows certain degree of localization for these processes in the interaction space. In resonance systems of such devices, it is possible to distinguish the elements of one electrodynamic process. In fact, the eigenfrequency spectrum corresponds to certain radio-frequency field's distribution and describes electrodynamic configuration of the resonance system. Knowing these fields and their space symmetry in advance it is possible to construct by decomposition a multipole generator model. As the system possesses resonance properties at the marked poles end, and as it is regenerated, it is presented as a system of mutual coupled oscillators. In turn, it is known that existence and stability of coherent oscillations in the systems of mutual coupled oscillators depend mainly on the mutual couplings configuration and their behavior. These issues are researched in the theory and practice of coherent systems [5–7].

Based on physics analogy, it is possible to insert "internal" mutual couplings between oscillating components, which model a real mechanism of oscillation stability. "External" mutual couplings can be introduced into the system in a similar manner. Unlike the internal ones, they are presented by real circuits.

Thus, we suppose that on the basis of the above statements and using azimuthal symmetry of magnetron fields, it is possible to distinguish a some pair of poles of the resonance system. Then the generator can be presented as on Fig. 1.

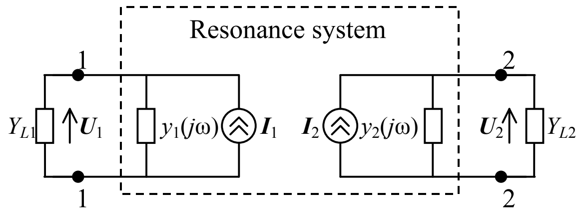

Fig. 1. Circuit of generator

Complex conductivity $y_k(j\omega)$ describe the properties of the oscillating system at poles 1 and 2, and resonance frequencies defined by them correspond to the oscillation mode. Conductivity Y_{Lk} in relation to the oscillating system are external, and they load it. We suppose that loaded resonance system on the frequency operating mode oscillations is characterized with relatively high selective properties. That is why almost harmonic oscillating process with ω_0 frequency builds up in this system; this process is described at terminals 1 and 2 by slowly changing complex voltage amplitudes

$$\boldsymbol{U}_k = U_k(t)e^{j\varphi_k(t)},\ k = 1,\ 2. \qquad (1)$$

Resonance system sections marked on the scheme (further on referred to as oscillating subsystems) are powered by induced currents. We suppose that only first current harmonics participate in the oscillating process. They are described by complex amplitudes $\boldsymbol{I}_k$. Phase difference takes place between the induced currents and voltages, that is why

$$\boldsymbol{I}_k = \left(-I_k^{re} + jI_k^{im}\right)e^{j\varphi_k(t)}. \qquad (2)$$

Then system's differential equations can be presented in a symbolic form

$$\boldsymbol{I}_k(t) + D(p)\boldsymbol{U}_k(t) = 0,\ p = \frac{d}{dt}. \quad (3)$$

According to the formal description of the slowly changing amplitudes method in its symbolic interpretation [5], the symbolic impedance $D(p)$ can be presented in approximated (abridged) form

$$D_k(p) = Y_{Lk} + j2C_k(\omega_0 - \omega_k) + 2C_k p, \quad (4)$$

where $C_k = \frac{1}{2}\frac{d(\operatorname{Im} Y_k(j\omega))}{d\omega}\Big|_{\omega_0}$. C_k parameters are proportionate to slope of phase characteristics of subsystems and analogous to tank circuit capacitance. Frequency properties Y_{Lk} are paid no regard to.

The distinguished oscillating subsystems are non-autonomous, they are parts of initial generating system and are interconnected through an electron flow. It means that the currents in (3) depend on all variables: amplitudes U_k and phases φ_k. As seen from (4), the model allows differences in resonance frequencies ω_k of the subsystems due to certain asymmetry of generator's oscillating configuration in relation to poles 1 and 2. Thus, the differential equations (3) describe coherent processes in the generator, presenting it as a system with two degrees of freedom.

3. Nonlinear Properties of the Model

When researching local motion as part of self-oscillatory systems' steady-state conditions problem, it is essential to perform linearization of the initial nonlinear differential equations. That is why it is necessary to determine functional relationships $\boldsymbol{I}_k(U_1, U_2, \varphi_1, \varphi_2)$, which reflect the connection of oscillations by means of the interaction between electromagnetic field and electron flow. We will construct the functions $\boldsymbol{I}_k$, basing on self-oscillating systems' common properties; phenomenological approach will be used. In-phase component of the current I_k^{re} with voltage $\boldsymbol{U}_k$ determines the power supplied to the field by the electron stream. Limiting properties of the system are defined by the function $I_k^{re}(U_k)$. Quadrature component causes the detuning of the oscillation frequency in relation to resonance frequency. Dephasing of current and voltage oscillations (i.e. $I_k^{im} \neq 0$) leads to processes' non-isochronism, which should be reflected by the functional relationship $I_k^{im}(U_k)$. Also, it should be taken into account that as a result of oscillating components interaction a mechanism for detention of stationary difference in phases $\Delta\varphi_0 = \varphi_{20} - \varphi_{10}$ is formed in the system. In this regard one can affirm that optimal interaction takes place in a well phased system. If for some reason phase difference disturbance occurs in the system, the amplitudes of self-oscillations should be changed, as well as coherent process frequency. The above stated implies that phase difference $\Delta\varphi = \varphi_2 - \varphi_1$ should be included in the parameters list of functions of the current.

Let us rewrite the abridged differential equations (3):

$$\begin{aligned}&\left[Y_k(U_k, U_l, \varphi_k - \varphi_l) + Y_{Lk} + \right.\\ &\left.+2C_k(\omega_0 - \omega_k) + 2C_k p\right]\boldsymbol{U}_k = 0, \quad (5)\\ &k = 1,\ 2;\ l \neq k,\end{aligned}$$

where $Y_k = \frac{\boldsymbol{I}_k}{\boldsymbol{U}_k} = -G_k + jB_k\ (G_k > 0)$ – complex conductance of active elements.

With the purpose of further formalization of the model (5) we will subject it to a permutation symmetry requirement, i.e. assume that the oscillating subsystems in question are characterized with qualitative identity. Then the assumption related to quantitative identity of linear and nonlinear parameters of subsystems offers us an optimal variant of adjustment and phasing: $\Delta\varphi_0 = \varphi_{20} - \varphi_{10} = 0$, $U_1 = U_2$, $\omega_0 = \omega_1 = \omega_2$, $B_k(\Delta\varphi_0 = 0) = 0$.

On the basis of this ideal variant let us determine the functional relationships profiles $B_k(\Delta\varphi)$, $G_k(\Delta\varphi)$. Steady-state equations (assuming that $p = 0$ in (5)) imply that the B_k functions are functions with alternating signs in terms of $\Delta\varphi$. On the other hand, deviation of the phase difference from its optimal value $\Delta\varphi_0=0$ is equivalent to the disturbance of field in the interaction space. This deteriorates the energy exchange with electron flow and decreases the regeneration degree. That is why functional relationship $G_k(\Delta\varphi)$ is presented as an even function with a maximum when $\Delta\varphi_0 = 0$.

Negative slope of the functional relationship $G_k(U_k)$ is obvious. It is determined by the limiting factors of the active environment, which is usually considered executed by default in oscillation tasks.

The function $G_k(U_l)$ $(l \neq k)$ is characterized with a reverse and smaller slope. It is proven on account of steady-state stability under amplitude disturbance. Finally, the profiles $B_k(U_k, U_l)$ define the direction of the gain-phase conversion, i.e. influence of the amplitudes on synchronous oscillations frequency ω_0. At this point it is natural to assume that under the force (non-parametric) interaction the frequency is pulled by more powerful self-oscillations. Application of the permutation symmetry provides the desired profiles.

The nonlinear model of a coherent system with two degrees of freedom presented in this report is qualitative. However, universal properties of self-oscillatory properties were used in the process of its construction.

4. Local Stability

As part of the local stability research we applied a standard procedure for differential equations linearization (5) and further substitution of solutions of

$\delta a_k^* \exp(\lambda t)$ type. As a result we got a system of algebraic equations for amplitude δa_k^* and phase $\delta\varphi_k^*$ disturbance:

$$\begin{bmatrix} 2C\lambda+\sigma & -\sigma_U^{re} & -\sigma_\varphi^{re} & \sigma_\varphi^{re} \\ -\sigma_U^{re} & 2C\lambda+\sigma & -\sigma_\varphi^{re} & \sigma_\varphi^{re} \\ \sigma_U^{im} & -\sigma_U^{im} & 2C\lambda+\sigma_\varphi^{im} & -\sigma_\varphi^{im} \\ \sigma_U^{im} & -\sigma_U^{im} & -\sigma_\varphi^{im} & 2C\lambda+\sigma_\varphi^{im} \end{bmatrix} \times \begin{bmatrix} \delta a_1^* \\ \delta a_2^* \\ \delta\varphi_1^* \\ \delta\varphi_2^* \end{bmatrix} = 0 \quad (6)$$

The coefficients of the matrix result from linearization of nonlinear functions G_k (index *re*) and B_k (index *im*), and their signature corresponds to nonlinear properties of the model for $\Delta\varphi_0 > 0$. For $\Delta\varphi_0 < 0$ the signs before the coefficients σ_φ^{re} and σ_U^{im} should be changed to opposite. Compatibility condition for the system of equations (6) provides characteristic equation det $[A(\lambda)] = 0$, and its roots define the stationary mode stability.

In the general case, oscillating subsystems are not identical, and corresponding coefficients in the equivalent matrix positions (6) differ in value. In the assigned task the evaluation of assumptions in terms of stability made while constructing the model is of the greatest interest. That is why both variational equations (6) and further analysis are limited by the case of identity of all subsystem's parameters except frequencies (i.e. $\omega_1 \neq \omega_2$, $\Delta\varphi_0 \neq 0$).

As is well known, finding roots of a characteristic equation and corresponding solution vectors make up a full eigenvalue problem [8]. The eigenvalues describe the nature of local motion, and vectors – directions in the phase space. For systems whose characteristic matrix possesses symmetry elements, it is possible to change the normal order of solving the problem and determine the eigenvectors first [6]. Afterwards the corresponding values can be found by a simple substitution of vectors into the equations (6). The simplications made allow applying this approach to our problem. Let us copy out the eigenvectors and eigenvalues:

1. $\delta a_1^* = \delta a_2^* \neq 0$, $\delta\varphi_1^* = \delta\varphi_2^* = 0$, $2\tilde{N}\lambda_1 = -\sigma + \sigma_U^{re} < 0$;
2. $\delta a_1^* = \delta a_2^* = 0$, $\delta\varphi_1^* = \delta\varphi_2^* \neq 0$, $2\tilde{N}\lambda_2 = 0$;
3. $\delta a_1^* = \delta a_2^* \neq 0$, $\delta\varphi_1^* = -\delta\varphi_2^*$, $2\tilde{N}\lambda_3 = -2\sigma_\varphi^{im} < 0$;
4. $\delta a_1^* = -\delta a_2^*$, $\delta\varphi_1^* = \delta\varphi_2^* \neq 0$, $2\tilde{N}\lambda_4 = -\sigma - \sigma_U^{re} < 0$.

The solutions obtained have clear physical meaning. The system is tested for stability of stationary amplitudes by their identical disturbances (solution 1). As seen, the condition mentioned above – $\sigma > \sigma_U^{re}$ – results from the stability requirement. If assumed that the subsystems in question are not coupled ($\sigma_U^{re} \equiv 0$), then the inequation 1 presented in this manner is known as amplitude stability requirement for an isolated generator, and coefficient $\sigma = -U_0 dG/dU|_0$ is referred to as limiting cycle strength. According to the solution 2, the system does not react to one-direction phase disturbance, which reflects the initial phase uncertainty characterized for generating systems. Stability to reverse phase disturbances (solution 3) is predefined by the properties of current functions describing the interaction of model's oscillating subsystems. Finally, the system remains stable under opposite amplitude disturbance.

Thus, the performed analysis proves that the self-oscillating model of a magnetron constructed using the phenomenological approach, adequately describes the interaction of the distinguished subsystems for stable coherent modes.

6. The System with External Coupling

Let us insert an external coupling by joining the poles of subsystems 1 and 2 (Fig. 1) through a symmetrical passive four-pole. The abridged equations of such model are obtained directly from (6), by replacing Y_{Lk} in them:

$$Y_{L1} = Y_{11} + Y_{12}\frac{\boldsymbol{U}_2}{\boldsymbol{U}_1},$$

$$Y_{L2} = Y_{22} + Y_{21}\frac{\boldsymbol{U}_1}{\boldsymbol{U}_2},$$

where $Y_{11} = Y_{22}$, $Y_{12} = Y_{21} = -g\exp(j\alpha)$ $(g > 0)$ – parameters of conductivity matrix of the four-pole. The g parameter defines the external coupling value, and α – its phase properties.

The new modified structure is analogous to the system of two mutual coupled oscillators in terms of physical and mathematical models. The mutual synchronization theory shows that resistance coupling of the first type ($\alpha = 0$) is optimal for stability of oscillations similar to in-phase oscillations [5–7]. In practice the selection of such coupling is performed by alternating the phase parameter of the communication circuit; it is the most essential element of the adjustment process of microwave range coherent systems.

In according to problem linearity, each element of the new matrix is presented as a sum of components describing internal and external coupling. As the problem structure is analogous to (6), its solution for eigenvalues can be written by analogy with the previous one.

1. If the system is tested with identical amplitude disturbances ($\delta a_1^* = \delta a_2^* \neq 0$, $\delta\varphi_1^* = \delta\varphi_2^* = 0$), then the reaction of coupling under such motion is excluded, and stability requirements $2\tilde{N}\lambda_1 = -\sigma + \sigma_U^{re} < 0$ fully match the analogous ones for a system without external coupling.

2. The system does not react to identical phase disturbances ($\delta a_1^* = \delta a_2^* = 0$, $\delta\varphi_1^* = \delta\varphi_2^* \neq 0$) as well: $2\tilde{N}\lambda_2 = 0$.

The influence of the external coupling becomes apparent when the system is disturbed in transversal directions of the phase space that correspond to the opposite variations of phases and amplitudes.

3. In case of stationary phase difference disturbance ($\delta a_1^* = \delta a_2^* \neq 0$, $\delta\varphi_1^* = -\delta\varphi_2^*$) local stability is characterized with a following requirement: $2\tilde{N}\lambda_3 = -2\left(\sigma_\varphi^{im} + g\cos\Delta\varphi_0\right) < 0$.

We can see that in case of resistance coupling of the first type ($\alpha = 0$), when $Y_{12} = -g$, the in-phase mode or similar modes degree of stability increases. Further the coupling of this kind will be referred to as favorable. On the contrary, for resistance coupling of the second type ($\alpha = \pi$), when $Y_{12} = +g$ (in this case the sign before **g** should be changed to opposite), decrease in stability is observed. In case of strong unfavorable interaction the operating coherent mode can lose its stability ($g > \sigma_\varphi^{im}$).

The second type of resistance coupling is favorable for the mode of antiphased ($\Delta\varphi_0 = \pi$) or similar oscillations.

4. Reciprocal amplitude disturbance ($\delta a_1^* = -\delta a_2^*$, $\delta\varphi_1^* = \delta\varphi_2^* \neq 0$) is damped by the system by means of both amplitude stability mechanism and external coupling:

$$2\tilde{N}\lambda_4 = -\sigma - \sigma_U^{re} - 2g\cos\Delta\varphi_0 < 0 .$$

Thus, we have showed that external coupling of generators oscillating subsystems has a great impact on local motion near to stationary state. The parameters of external coupling channels g and α correspond to the primary parameters of actual four-poles and have no fundamental restrictions for their alteration. At the same time in the course of experiment they are fully controlled and can be adjusted quite accurately. The eigenvalues determine the disturbances' relaxation rate, therefore it can be affirmed that favorable external coupling strengthens the internal (electronic) mechanism of phase relations detention. Experimental research of relativistic magnetron shows that introduction of external couplings into its resonance system improves the process of energy exchange and significantly increases spectral and mode stability of oscillations [9, 10].

7. Conclusion

In this report a self-oscillatory model of a magnetron with distinguished oscillating subsystems was presented. Using the phenomenological approach an adequate description of subsystems' interactions for stable coherent modes was given. For solving the stability problem, an apparatus of full eigenvalue problem was applied. In case of acceptable simplification of the model it provides a full picture of local motions and mechanism of interactions' influence on stability. It is shown that introducing external coupling can increase the coherent processes' degree of stability. The theoretical model developed in the report can be applied for modification of properties of microwave range generating devices.

References

[1] T.A. Treado, W.O. Dogget, E.E. Thomas et al., IEEE Trans. **PS-16**, 2, 237 (1988).

[2] I. Schitzer, A. Rosenberg, C. Leibovitz et al., Proc. of SPIE **2843**, 101 (1995).

[3] N.P. Gadetski, I.I. Magda, S.I. Neisteter, Plasma Phys. Rep. (Sov.) **19**, 4, 273 (1993).

[4] S.A. Kitsanov, A.I. Klimov, S.D. Korovin et al., *in: Proc. 1st Int. Congress on Radiation Physics, High Current Electronics, and Modification of Materials, Tomsk*, 2000, **2**, pp. 423–428.

[5] A.A. Dvornikov, G.M. Utkin, *Phased self-oscillator of Radio transmitting equipment*, Moscow, Energy, 1980, 177 pp.

[6] S.N. Vladimirov, A.S. Maidanovskii, S.S. Novikov, *The nonlinear oscillations of multifrequency autooscillators,* Tomsk, Tomsk University Publishing House, 1993, 203 pp.

[7] S.A. Maidanovskii, S.S. Novikov, Radiotechnika i electronica **48**/6, 1 (2003).

[8] D.R. Merkin *Introduction in theory of motion stability*, Moscow, Nauka, 1971, 312 pp.

[9] I.I. Vintizenko, A.I. Zarevich, S.S. Novikov, "Microwave Radiation Characteristics of Relativistic Magnetron with Coupled Cavities", presented at *13th Symposium on High Current Electponics*, Tomsk, Russia, 2004.

[10] I.I. Vintizenko, A.I. Zarevich, S.S. Novikov, "Distributed Output of Microwave Radiation from Relativistic Magnetron", presented at *13th Symposium on High Current Electronics*, Tomsk, Russia, 2004.