MAGIC 2D Simulation of Nonstationary and Chaotic Processes in a Relativistic Backward Wave Oscillator[1]

N.M. Ryskin, V.N. Titov, Y.-B. Kang*, G.-S. Park*

Saratov State University, Astrakhanskaya str. 83, Saratov 410012, Russia, tel.: 7(8452)514311,
e-mail: RyskinNM@info.sgu.ru
** School of Physics, Seoul National University, Seoul 151-742, Korea*

Abstract – The results of detailed study of self-modulation and chaos in a relativistic backward wave oscillator obtained by MAGIC 2D numerical simulation are presented. A complex sequence of self-modulation regimes based on different axial modes is observed with the increase of a beam current. Various transitions to chaos are described. The results are compared with previous studies based on simpler 1D non-stationary code as well as with existing experimental results.

1. Introduction

Study of non-stationary and transient processes is of major importance for the development of relativistic backward wave oscillators (BWOs). Such phenomena have been addressed by several works [1–17], both theoretically and experimentally. The dynamics of a relativistic BWO is strongly affected by reflections of radiation from the ends of a slow wave structure (SWS). It was revealed [3, 5, 6, 8, 13, 17] that the mechanism of a self-modulation and chaotization at strong reflections (resonant BWO) differs considerably from the case of small reflections (non- resonant BWO). In the former, the processes are basically defined by the competition of axial modes of a resonant structure. Self-modulation is caused by hard (subcritical) excitation of one of the high-order axial modes (so called cross-excitation instability [6] or frequency mechanism of self-modulation, see [18–20] for details). The frequency mechanism is also typical for traveling wave tube (TWT) delayed feedback oscillators [18–19], free electron lasers [20, 21], and optical parametric oscillators [22]. On the contrary, a non resonant BWO is characterized by soft excitation of self-modulation [1, 9–13] (so called overbunch instability [6] or amplitude mechanism [18–20]) and by the transition to chaos through period doubling scenario [2, 9–12]. Similar behavior is also typical for klystron delayed feedback oscillators [23], gyro BWO [24], and TWT oscillators with a narrow band resonant filter included in the feedback loop [25].

However, in the most of works nonlinear dynamics of BWO is studied using a simple 1D nonstationary code based on slowly varying envelope approximation [1–3, 5, 6, 9–16], supposing reflections to be frequency independent. In an actual device, however, the reflections may depend strongly on frequency. Correct account for the frequency dependence is especially important in the case of broadband multifrequency and chaotic oscillations, when for some spectral components the reflection coefficient can be much greater then for the others. As a result, a complicated combination of resonant and non-resonant scenarios can be observed.

2. Numerical Model

This paper is dedicated to the numerical modeling of transition to chaos in the relativistic BWO by MAGIC2D code [26] and comparison with the results of numerical [6, 9–13] and experimental [14–16] works. MAGIC is a self-consistent finite difference electromagnetic particle-in-cell (PIC) code allowing simulation of realistic slow wave structures and naturally taking into account various important phenomena, such as effects of transverse electron motion and beam interaction with a forward wave reflected from the cutoff neck, which may lead to serious improvement of BWO performance [8]. The nonequidistant character of the frequency spectrum of axial modes near cutoff [17] is also naturally included in a simulation.

For the simulation, we selected the parameters of the BWO, described in [14–16], where chaotic generation of a MW power in a weakly relativistic BWO was achieved experimentally for the first time. The distinctive feature of this BWO is the usage of thermionic electron injector [27], that allows to obtain a current pulse with duration up to 10 ms. In the case of traditional relativistic carcinotrons with high-current nanosecond pulse [4] transient time is comparable to pulse length, so it is impossible to investigate chaotic phenomena in the sense of nonlinear dynamics, where chaos is associated with a strange attractor in a phase space.

In this case one can study complex dynamics of the transient processes and mode competition phenomena during the onset of a current pulse shape [5, 6, 17]. However, strictly speaking, such phenomena cannot be classified as deterministic chaos.

---

[1] This work is partly supported by CRDF Award No. REC-006 and by the FTP “Integration” grant No. A0057. V.N. Titov’s work is also supported by the grant of President of Russian Federation for young scientists No. MK 26.2003.02.

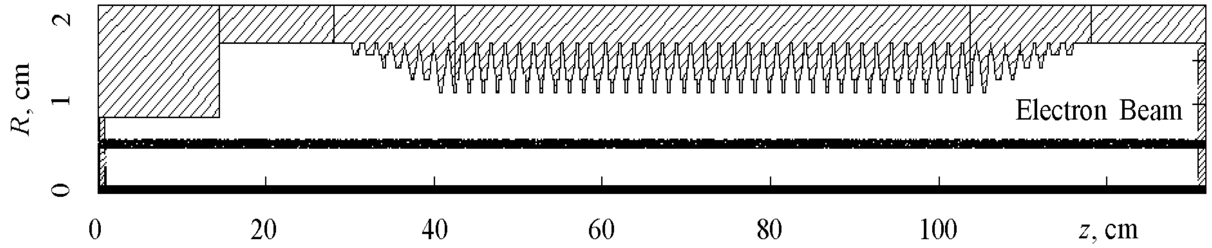

Fig. 1. Schematic drawing of the electrodynamic structure used in simulations

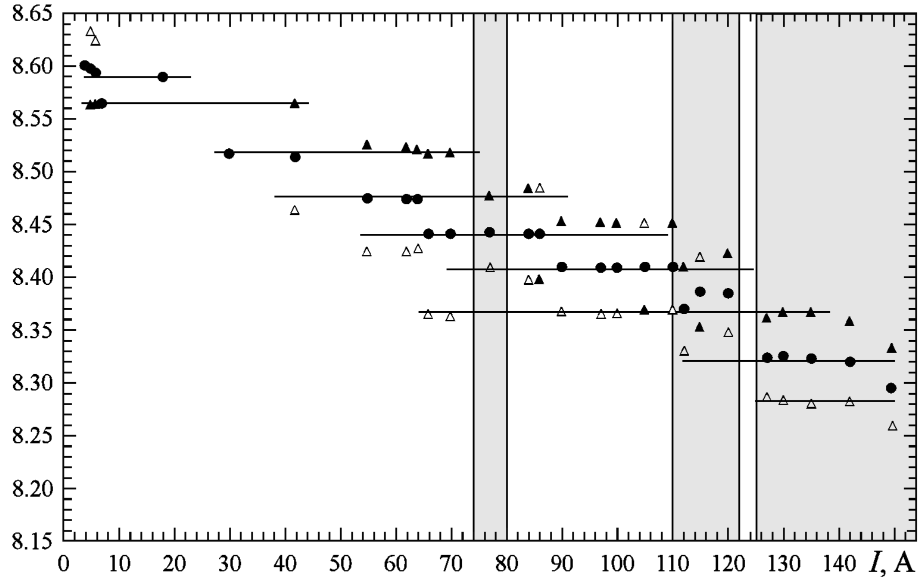

Fig. 2. Frequencies of main spectral components versus beam current. Fundamental frequency is denoted by circles and main sidebands by triangles. Black triangles mark dominating sideband. Regions of chaos are shaded

According to [14–16] we chose the length of the rippled wall SWS to be 62.3 cm, the mean waveguide radius 1.4 cm, the period and corrugation amplitude 1.7 and 0.25 cm, respectively, the electron beam radius 0.6 cm, and beam voltage 150 kV. To minimize the reflections we included two tapered sections of a moderate length with adiabatic decrease of the corrugation amplitude. For the cutoff neck we selected the radius of 0.8 cm. Schematic of the device used in simulations is shown on Fig. 1.

3. Results and Discussion

We simulated the sequence of dynamical regimes taking place with the increase of a beam current I. The general pattern of the dynamics is shown in Fig. 2, where the fundamental frequency and main sidebands are plotted versus beam current. It is clearly visible that spectral components exhibit hopping variations. This confirms the principal role of the end reflections despite the presence of tapering sections.

We also performed variation of the length of cylindrical section between the cutoff neck and the rippled waveguide that is approximately equivalent to the phase shift of the reflection coefficient in terms of [13]. Although the general pattern remains intact, the actual sequence of transitions may alter. The slight displacement of the cutoff neck affects both resonant frequencies and bifurcation values of current, corresponding to transitions between regimes based on different resonant modes. This conclusion agrees with the results of [8, 13], where it was shown that the phase of reflection coefficient considerably affects the system operation. As the displacement of cutoff neck strongly affects the dynamics, an optimization of a relativistic BWO operation by the appropriate choice of the cutoff neck geometry should be possible.

We wish to comment some general observations we made. The self-excitation occurs at $I \approx 3$ A. The generation frequency is about 8.6 GHz, which is a bit lower than that observed in experimental works [14–16] and practically does not depend on a distance between a cutoff and SWS.

Self-modulation already emerges at $I \approx 6$ A, corresponding to unusually small current to starting current ratio $I/I_{\mathrm{st}} \sim 2$, whereas in non-resonant BWO $I/I_{\mathrm{st}} \sim 3.14$ [1, 9].

The self-modulation arises due to the frequency mechanism, i.e. by the hard excitation of an axial mode nearest to the fundamental frequency. This self-modulation regime exists only in a very narrow range of the beam current. Already at $I = 7$ A a resonant axial mode wins during a mode competition, leading to the restoration of single frequency generation with carrier frequency hoping to the frequency of resonant mode. Actually, according to [6, 13] in resonant BWO the self-modulation threshold can be very close to the self-excitation threshold. Besides, with the subsequent increase of the beam current self-modulation can be replaced by single frequency generation (see Fig. 14 in [6]).

At $I \approx 40$ A self-modulation reappears again, but this time due to the amplitude mechanism. In the spectrum we observe two symmetric satellites with almost equal amplitude that differs strongly from spectra corresponding to frequency mechanism. Self-modulation frequency increases distinctly as well. Actually, in the case of amplitude mechanism one can estimate the characteristic time of internal feedback as $T = l/v_0 + l/v_g$, where v_0 is beam velocity, v_g is slow wave group velocity, and l is length of the SWS [1, 11, 13]. Period of self-modulation is known to be about 1.5 T [1, 9–11]. This rough estimation is in a good qualitative agreement with simulation results (57 MHz and 51 MHz from simulation at $I = 42$ A).

With the increase of a beam current spectral components are shifting downwards until they are locked by axial modes, similarly to the picture described in [13]. Being locked, the frequencies remain constant until the transition to the modes with lower frequencies takes place. As a result, stepwise frequency shift is observed.

Gradually, the spectrum starts to exhibit features typical of the frequency mechanism, with the right sideband having maximum amplitude, positioned exactly at the frequency of the mode, which was dominant before. On the contrary, low frequency satellites always dominate in the spectrum of a non-resonant BWO [11].

Our results are in a qualitative agreement with experimental spectra, presented in [15, 16], where at low beam currents left-hand satellites dominate, and at higher current right-hand ones, i.e. the gradual transition from the non-resonant to resonant mechanism takes place. Thus, the basic features of experimental spectra of [15, 16] can be explained by the influence of reflections, rather than by the effects of a space charge, as it was suggested in corresponding papers.

At $I = 65–70$ A axial modes nearest to the fundamental are suppressed during the mode competition and sidebands are formed by higher-order modes. Consequently, the self-modulation frequency abruptly rises approximately twice. With the subsequent increase of the beam current, the nearest modes are excited again and the frequency of a self-modulation decreases twice. Such behavior was reported in [13] . Some of experimental spectra in [15, 16] also indicate the presence of similar phenomena.

A transition from one mode to another can occur by two scenarios: either by hard transitions as a result of a subcritical bifurcation (mode hopping), or via chaotic intermittency, i.e. the irregular beating of different modes. This is also in a good agreement with the results of [13]. However, intermittency takes place only in a narrow range of a beam current, and it is subsequently replaced by a periodic or quasiperiodic self-modulation with a discrete spectrum. Thus, with the increase of the beam current one observes not a single transition to chaos, but a complex sequence of alternating regular (single frequency or multi-frequency with discrete spectrum) and chaotic oscillation regimes. This is a feature of beam-wave electronic systems and, widely speaking, of spatially extended multimode systems [9–13, 23].

Above $I = 110$ A, oscillations are basically chaotic with a continuous spectrum. However, the discrete spectral components on the frequencies of “cold” axial modes are clearly observable (see for example Fig. 3). This pattern is in a good agreement with that of [13] , and also with experimental spectra reported in [15, 16]. With the increase of a beam current, spectral components occasionally exhibit stepwise shift downwards, thus indicating successive transitions to regimes based on consecutively lower frequency modes. On the contrary, spectra of non-resonant BWO are much more homogeneous (see the numerical results in [15, 16]) while the spectral components exhibit monotonous downshift.

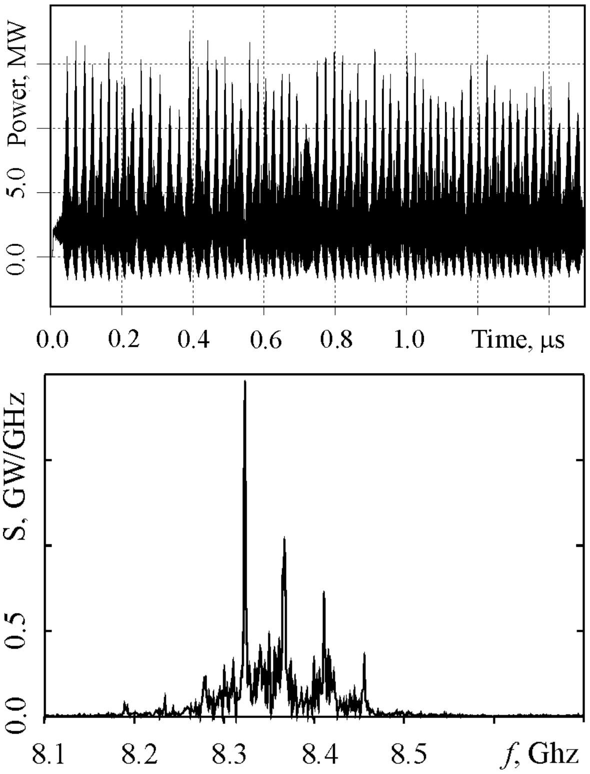

Fig. 3. Output power waveform and corresponding spectrum at $I = 135$ A

4. Conclusion

In conclusion, the results of detailed study of self-modulation and chaos in relativistic BWO obtained by MAGIC 2D simulation are presented for the first time. It is shown that the basic pattern of the dynamics revealed earlier in [13] by simple 1D nonstationary simulation remains valid. In particular, with the increase of the beam current the complex sequence of self-modulation regimes based on different axial modes is observed. The transitions between different regimes occur through hard excitation or via intermittency. Even at large current to starting current ratio the discrete spectral components on the frequencies of "cold" axial modes are clearly visible on the background of low noise pedestal.

However Magic 2d code takes into account such important factors, as the frequency dependence of reflection from the ends of actual SWS, transversal effects, interaction of electron beam with a reflected forward wave etc. As a result, the pattern of dynamics differs in some extent from that described in [13] , more precisely; there is a combination of resonant and non-resonant scenarios. Our results are also in a good agreement with experimental works [14–16] and emphasize the importance of reflections in the dynamics of a relativistic BWO. However, only qualitative comparison is valid, since we do not have exact specification concerning the area of tapering and cutoff neck of the actual device used in the experiment, the uncertainty, which may affect the operation, particularly the nonstationary regimes.

References

[1] N.S. Ginzburg, S.P. Kuznetsov, T.N. Fedoseeva, Izv. Vyssh. Uchebn. Zaved. Radiofiz. **21**, 1037 (1978) [Radiophys. Quant. Electron. **21**, 728 (1979)].

[2] A.O. Ostrovskii, Yu.V. Tkach, Pis'ma v Zh. Tekh. Fiz. **17** (18), 10 (1991) [Sov. Tech. Phys. Lett. **17**, 645 (1991)].

[3] V.A. Bondarenko, A.O. Ostrovskii, Y.V. Tkach, Zh. Tekh. Fiz. **60**, 134 (1990).

[4] Y. Carmel, W.R. Lou, J. Rodgers, H. Guo, W.W. Destler, V.L. Granatstein, B. Levush, T. Antonsen, A. Bromborsky, Phys. Rev. Lett. **69**, 1652 (1992).

[5] B. Levush, T.M. Antonsen, A. Bromborsky, W.R. Lou, Y. Carmel, Phys. Fluids **B4**, 2293 (1992).

[6] B. Levush, T.M. Antonsen, A. Bromborsky, W.R. Lou, Y. Carmel, IEEE Trans. Plasma Sci. **20**, 263, (1992).

[7] I.V. Pegel, Russian Physics Journal **39**, 1210 (1996).

[8] L.D. Moreland, E. Schamiloglu, R.W. Lemke, A.M. Roitman, S.D. Korovin, V.V. Rostov, IEEE Trans. Plasma Sci. **24**, 852 (1996).

[9] N.M. Ryskin, V.N. Titov, D.I. Trubetskov, Dokl. Ross. Akad. Nauk **358**, 620 (1998) [Doklady Physics **43**, 90 (1998)].

[10] N.M. Ryskin, V.N. Titov, Izv. Vyssh. Uchebn. Zaved. Appl. Nonlinear Dynamics **6** (1), 75 (1998).

[11] N.M. Ryskin, V.N. Titov, Izv. Vyssh. Uchebn. Zaved. Radiofiz. **42**, 566 (1999) [Radiophys. Quant. Electron. **42**, 500 (1999)].

[12] N.M. Ryskin, V.N. Titov, Journ. Commun. Technol. Electronics **45**, S46 (2000).

[13] N.M. Ryskin, V.N. Titov, Izv. Vyssh. Uchebn. Zaved. Radiofiz. **44**, 860 (2001) [Radiophys. Quant. Electron. **44**, 793 (2001)].

[14] N.S. Ginzburg, N.I. Zaitsev, E.V. Ilyakov, I.S. Kulagin, Yu.V. Novozhilova, A.S. Sergeev, Izv. Vyssh. Uchebn. Zaved. Applied Nonlinear Dynamics **7** (5), 60 (1999).

[15] N.S. Ginzburg, N.I. Zaitsev, E.V. Ilyakov, I.S. Kulagin, Y.V. Novozhilova, R.M. Rozental, A.S. Sergeev, Zh. Tekh. Fiz. **71** (11), 73 (2001) [Technical Physics **46**, 1420 (2001)].

[16] N.S. Ginzburg, N.I. Zaitsev, E.V. Ilyakov, I.S. Kulagin, Yu.V. Novozhilova, R.M. Rozenthal, A.S. Sergeev, Phys. Rev. Lett. **98**, 108304 (2002).

[17] G.S. Nusinovich, Yu.P. Bliokh, Phys. Plasmas **7**, 1294 (2000).

[18] Yu.P. Bliokh, M.G. Liubarskii, V.O. Podobinskii, Ya.B. Fainberg, Fiz. Plazmy **20**, 718 (1994) [Plasma Phys. Rep. **20**, 648 (1994)].

[19] Yu.P. Bliokh, M.G. Liubarskii, V.O. Podobinskii, Ya.B. Fainberg, G.S. Nusinovich, S. Kobayashi, Y. Carmel, V.L. Granatstein, Phys. Plasmas **5**, 4061 (1998).

[20] T.M. Antonsen, B. Levush, Phys. Fluids **B1**, 1097 (1989).

[21] N.S. Ginzburg, M.I. Petelin, Int. Journ. Electronics **59**, 291 (1985).

[22] T.V. Dmitrieva, N.M. Ryskin, Zh. Eksp. Teor. Fiz. **120**, 1517 (2001) [JETP **93**, 1314 (2001)].

[23] B.S. Dmitriev, Y.D. Zharkov, N.M. Ryskin, A.M. Shigaev, Radiotekh. Electron. **46**, 604 (2001) [Journ. Comm. Technol. Electron. **46**, 561 (2001)].

[24] T.H. Chang, S.H. Chen, L.R. Barnett, K.R. Chu, Phys. Rev. Lett. **87**, 064802 (2001).

[25] V.A. Katz, Radiophys. Quant. Electron. **28**, 161 (1985).

[26] L. Ludeking, D. Smithe, M. Bettenhausen, and S. Hayes, *MAGIC User's manual*, Newington, Mission Research Corporation, 2003.

[27] E.V. Ilyakov, G.S. Korablyov, I.S. Kulagin, N.I. Zaitsev, IEEE Trans. Plasma Sci. **26**, 332 (1998).