Nonlinear Dynamics in a Free Electron Laser Amplifier with Electromagnetic Pumping[1]

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Abstract – Results of theoretical study of nonlinear dynamics in a model of scattron free electron laser (FEL) amplifier based on the induced backscattering of two transversal electromagnetic waves on a relativistic electron beam are considered. Effect of self-modulation of output radiation is studied numerically. The increase of either input power or beam current results in complication of self-modulation regime and arising of quasi-periodic or even chaotic oscillations. Basic properties of single-frequency, multi-frequency (self-modulational) and chaotic regimes are discussed. A bifurcation map on the parameter plane is presented.

1. Introduction and Basic Equations

Free electron lasers (FELs) have the potential of providing very high-power, continuously tunable, coherent radiation over an extensive range of wavelengths [1], and are currently the subject of intensive research effort. Usually FELs utilize periodic magnetostatic undulator and strongly relativistic electron beam to obtain high frequency up-conversion. However, high-power radiation may be obtained using moderately relativistic electron beams and electromagnetic wave pumping (scattron FEL). This may result in development of compact sources of THz radiation. For example, using a powerful millimeter wave generator as a pump source one needs about 10 times frequency conversion to obtain output radiation with teraherz frequencies. For this, beam voltage about 300 kV is required.

In this paper, we study nonlinear behavior of a FEL amplifier with electromagnetic pumping being focused on the nonstationary behavior. We consider the parametric interaction of a signal with frequency ω_s with slow space charge wave with frequency $\omega_i = \omega_s - \omega_p$ and the counter-propagating electromagnetic pump wave with frequency ω_p. We are investigating the case of a moderately relativistic electron beam ($\gamma_0 \le 2$), nevertheless providing sufficient frequency conversion. Our analysis is based on the nonlinear theory that has been developed earlier in [2–4]. In the dimensionless form the nonstationary equations are

$$\left(\frac{\partial}{\partial\xi}+\frac{c}{v_0}\frac{\partial}{\partial\tau}\right)^2\theta = -\mathrm{Re}\left(\frac{v_0^2L^2}{c^2}F_sF_p^*e^{i\theta}\right), \qquad (1)$$

$$\frac{\partial F_s}{\partial\xi}+\frac{\partial F_s}{\partial\tau} = -LF_pI\,, \qquad (2)$$

$$\frac{\partial F_p}{\partial\xi}-\frac{\partial F_p}{\partial\tau} = -\varepsilon LF_sI^*\,, \qquad (3)$$

where F_s and F_p are slowly varying amplitudes of the signal and pump waves, respectively; $I=\frac{1}{\pi}\int_0^{2\pi}\exp(-i\theta)d\theta_0$ is the amplitude of the first harmonic of bunched current; θ is the phase of the electrons, θ_0 – initial phase; ξ and τ are normalized coordinate and time, respectively; v_0 is the beam velocity. In the equations (1)–(3) there are three dimensionless parameters: the normalized length of the system L, relativistic mass-factor γ_0, and the parameter of pump depletion ε [4]. The pump wave is propagating counter to the electron beam and the signal wave.

The boundary conditions in the case of the amplifier are

$$I(0,\tau)=0\,,\ I_\xi(0,\tau)=0\,,$$

$$F_s(0,\tau)=F_0\exp(i\Omega\tau)\,,\ F_p(1,\tau)=1\,, \qquad (4)$$

where F_0 is the input signal amplitude and Ω is the normalized frequency shift from the exact parametric resonance frequency, ω_s.

Note, that in the limit $\varepsilon \to 0$ the eqs. (1)–(4) are equivalent to those describing a delayed feedback traveling wave tube (TWT) oscillator [5]. Eq. (1) describes motion of the electrons and is usually solved by Lagrange macro particles method [6]. In the theory of most vacuum electron devices such as TWT or BWO there exists an efficient technique allowing considering only the electrons that fit within one wave period (usually 32–64 items). However, this is possible only if electron velocity exceeds the group veloc-

[1] The work was supported by the grants of the Russian Found of Basic Research (No. 03-02-16269), Program "Universities of Russia – Basic Researches" (No. 01.01.49), and by the grant of Russian Ministry of Education for PhD students research support (No. A03-2.9-810).

ity of the signal. In our case, the interacting waves are fast waves with $v_g = c$, so the mentioned above approximation fails that leads to substantial complication of the numerical method.

2. Nonlinear Wave Model of a FEL Amplifier

In this section we consider a simplified model in the assumption that electron beam nonlinearity is weak, and the only significant nonlinear effect is the pump depletion. Thus, after linearization of (1) we obtain the system of nonlinear wave equations

$$\left(\frac{\partial}{\partial\xi}+\frac{c}{v_0}\frac{\partial}{\partial\tau}\right)^2 F_i = i\frac{\gamma_0^2-1}{\gamma_0^2}L^2 F_s F_p^*, \quad (5)$$

$$\frac{\partial F_s}{\partial\xi}+\frac{\partial F_s}{\partial\tau} = -LF_pF_i, \quad (6)$$

$$\frac{\partial F_p}{\partial\xi}-\frac{\partial F_p}{\partial\tau} = -\frac{\left(\gamma_0+\sqrt{\gamma_0^2-1}\right)^2}{\gamma_0^2}LF_sF_i^*, \quad (7)$$

where F_i is the normalized amplitude of the slow space-charge wave. Now only two control parameters, L and γ_0, remain in (4)–(6). The boundary conditions for the amplifier are

$$F_i(0) = \frac{\partial F_i(0)}{\partial\xi} = 0,\ F_s(0) = F_0 e^{i\Omega\tau},\ F_p(1) = 1, \quad (8)$$

where F_0 and Ω are amplitude and frequency of the input signal. The eqs. (5)–(7) were solved by an explicit finite-difference method of second order accuracy in space and time.

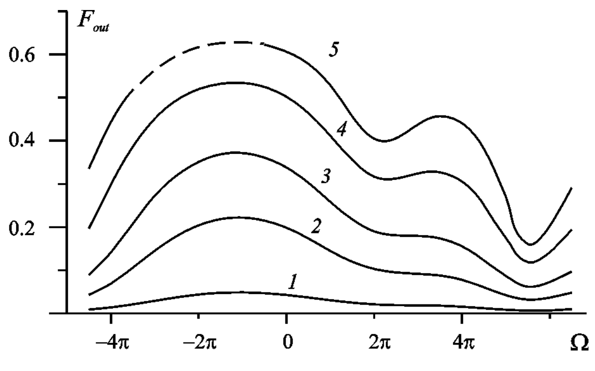

Fig. 1. The output signal amplitude as a function of frequency at $\gamma_0 = 1.3$, $L = 4$ and various amplitudes of the input signal: $F_0 = 0.01$ (1), 0.05 (2), 0.1 (3), 0.2 (4), 0.3 (5). Dashed line corresponds to a self-modulation regime

Equations (5)–(8) allow taking into account frequency dependence of the FEL gain. In Fig. 1 we present an example of frequency response of output signal amplitude for various input signal amplitudes. One can see that for large F_0 self-modulation arises near $\Omega = -1.5\pi$ shown by dashed line on the curve (5). The physical origin of self-modulation in the FEL amplifier is the parasitic feedback that takes place due to counter propagation of the pump wave [3, 4].

With the enlargement of γ_0 the threshold of self-modulation goes down. This can be explained as follows. The amplifier's bandwidth is determined by the ratio of the beam velocity v_0 to the signal velocity (that is assumed equal to the speed of light). The larger is γ_0 the wider is the bandwidth. Therefore, the gain at the self-modulation frequency increases.

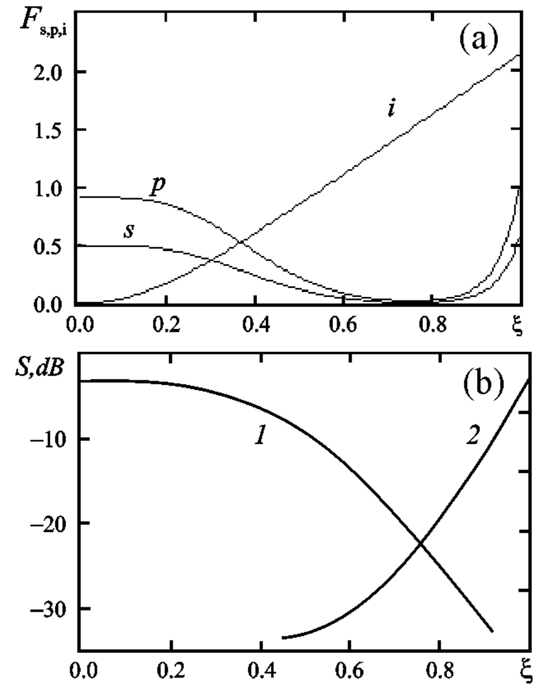

Fig. 2. (a) Spatial distributions of the wave amplitudes; (b) spectral intensities of the input signal (*1*) and the satellite (*2*); $\gamma_0 = 1.8$, $L = 5.5$, $\Omega = -\pi$, $F_0 = 0.5$

The region of self-modulation proves to be limited and with further enlargement of either L or F_0 the output signal becomes nearly single-frequency again. However, in the spectrum of the output signal the parasitic satellite dominates, not the input signal, which is almost completely suppressed. In Fig. 2(a) the distributions of the wave amplitudes along the system are presented. One can observe a domain where the signal is suppressed due to pump depletion, and the domain of rapid growth of the signal near the output. But this is the growth of the satellite, not of the input signal. This is confirmed by Fig. 2(b) where spectral intensities are plotted. The variables $F_{s,i,p}$ are almost time-independent, however in the region where the intensities of the satellite and the input signal are close ($\xi \sim 0.7$–0.8), small oscillations are observed.

The overall picture is illustrated by a map of dynamic regimes on the plane of parameters F_0 and L shown in Fig. 3. In this figure, the boundaries of self-modulation regime and the regime of nearly single frequency with the parasitic satellite being dominant are depicted.

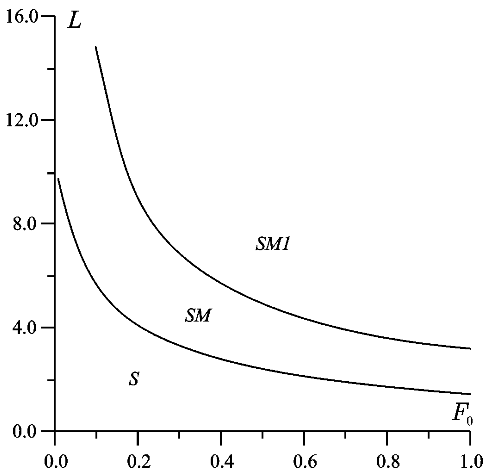

Fig. 3. The map of dynamic regimes on the plane of control parameters at $\gamma_0 = 1.8$, $\Omega = -\pi$, *S* is the stationary amplification regime, *SM* corresponds to self-modulation, *SM1* is nearly single frequency regime with parasitic satellite being dominant

3. Nonliear Dynamics of the FEL-Amplifier Considering Electron Beam Nonlinearity

Subsequently we studied a more complicated model of FEL amplifier (1)–(4) that takes into account electron overbunching effects. This required substantial complication of the numerical method to solve the equations of motion (1). At each point of the spatial grid we placed 32 or 64 "Lagrange particles" to be able to calculate the current harmonic I. Since the number or grid nodes is about 100–200, the total number of electrons is of order of several thousands. In comparison, in the theory of TWT or BWO, we should follow the motion of only 32–64 electrons.

In general, the results are qualitatively similar to those predicted by the model described in Sec. II. As an example, the gain–frequency response is presented in Fig. 4. However, self-modulation arises in that system for larger values of the input signal (cf. Fig. 1). Simulations show that with the enlargement of γ_0 as well as with the enlargement of pump depletion parameter ε the threshold of self-modulation goes down.

However, with the increase of the length parameter *L* more complicated dynamics is observed, including quasiperiodic and chaotic self-modulation. Thus, electron nonlinearity is vital for chaos. In Fig. 5 the map of dynamic regimes is presented. The quasi-periodic region proves to be narrow. Increasing of the bifurcation parameter results in quasi-periodic route to chaos.

Further behavior of the system may be different, depending on F_0. For not too large F_0 ($F_0 < 0.7$ for the case presented in Fig. 5), after the chaotic regime periodic self-modulation is restored with the input signal being almost completely suppressed. The situation is similar to that described in Sec. II (cf. Fig. 3). Note that the transition from chaos to the regular self-modulation occurs via intermittency.

Then one more quasiperiodic transition to chaos takes place. Now the dominating frequency is that of the parasitic satellite. Examples of phase portraits and power spectra illustrating that transition are presented in Fig. 6. In the spectrum of the output signal the parasitic satellite dominates, not the input signal, which is almost completely suppressed.

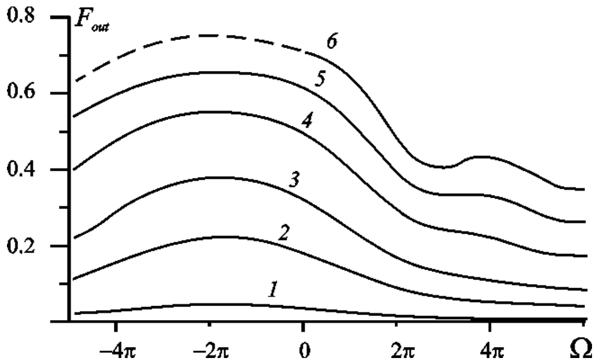

Fig. 4. The amplifier's output signal amplitude as a function of frequency at $\gamma_0 = 1.3$, $L = 4$, $\varepsilon = 2.5$ and various amplitudes of the input signal: $F_0 = 0.01$ (1), 0.05 (2), 0.1 (3), 0.2 (4), 0.3 (5), 0.4 (6). Dashed line corresponds to a self-modulation regime

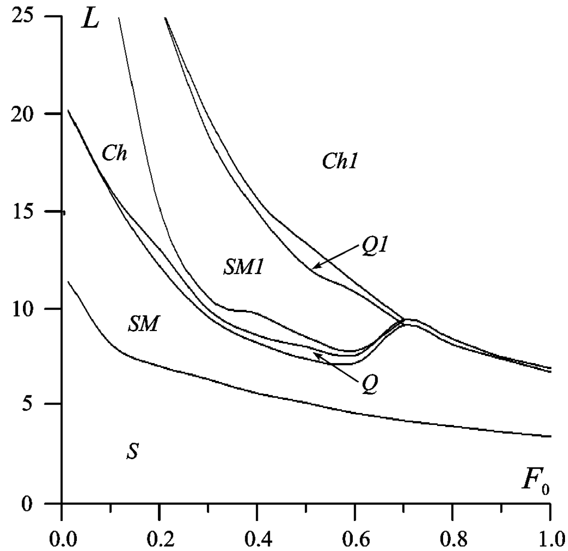

Fig. 5. The map of dynamic regimes on the plane of control parameters ($\gamma_0 = 1.8$, $\Omega = -\pi$, $\varepsilon = 0.4$): *S* – stationary amplification regime, *SM* – periodic self-modulation, *Q* – quasi-periodic self-modulation, *Ch* – chaos. The same designations with prefix "1" correspond to the regimes with parasitic satellite being dominant

For $F_0 > 0.7$ there are no quasi-periodic and chaotic regimes based on the input signal frequency. In the domain of periodic self-modulation, smooth transition from the regime based on the input frequency to that based on the satellite frequency takes place. Then quasi-periodic route to chaos, similar to that presented in Fig. 6, is observed.

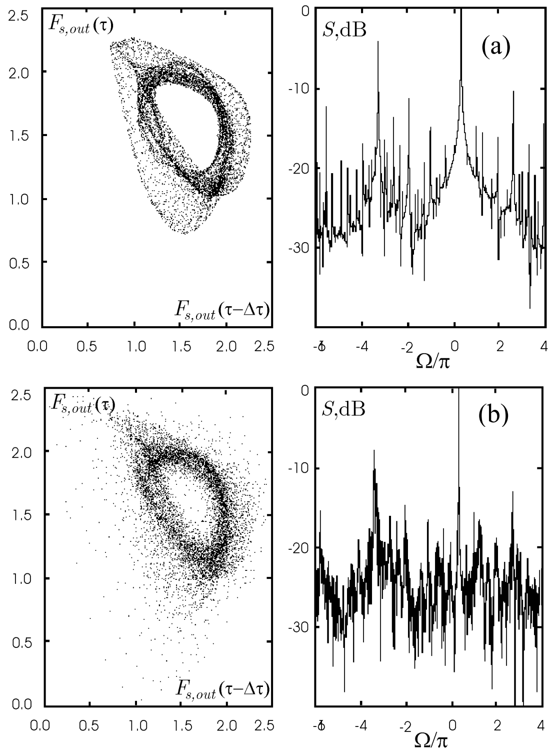

Fig. 6. Phase portraits and power spectra of the output signal at $\gamma_0 = 1.8$, $\Omega = -\pi$, $F_0 = 0.5$, in the quasi-periodic regime Q_1, $L = 13.2$ (a) and in the chaotic regime Ch_1, $L = 13.5$ (b). Note that main frequency in the spectrum differs from that of the input signal

4. Conclusion

Nonlinear dynamics of a scattron FEL amplifier based on the induced backscattering of two transversal electromagnetic waves on a relativistic electron beam is studied in detail. Two basic models are studied. The first one is based on the nonlinear wave equations obtained in the assumption that electron overbunching effects are negligible. The second is based on more rigorous equations taking into account electron nonlinearity. The main attention is paid to the effect of self-modulation of the output radiation. For the second model, with the increase of the control parameters more complicated quasi-periodic or even chaotic oscillations are possible. Transition to chaos occurs via a quasi-periodic route.

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