On the Virtual Cathode Velocity During the Transportation of an Electron Beam in Drift Tubes

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Abstract – The dependence of the virtual cathode velocity V on the current of the injected electron beam and on the current beyond virtual cathode have been determined by going from a coordinate system K where the virtual cathode is immobile and for which the steady states are known to a system K' which move with the same velocity as the virtual cathode. In so doing, use was made of Lorentz's transformations for the strengths of electric and magnetic fields and a maximum virtual cathode velocity has thus been found. Theoretical results agree with data obtained by numerical simulation.

1. Introduction

In [1–3] numerical calculations have shown that in certain conditions of electron beam injection into a two-stage drift channel a virtual cathode (VC) is formed near the stage joint. As the injection current approaches its critical value, the virtual cathode starts moving toward the injected electron beam, leaving behind it a pinched unidirectional flow [1–3]. One can use this phenomenon, e.g., for collective accelerations of ions [4]. In so doing, the energy of the ion bunch trapped by the potential well is determined by the VC velocity. In this work we examine the problem on the virtual cathode velocity in a drift channel with $R_1 = R_2$.

2. Theory

In [5] the problem on the transportation of a tubular electron beam in a drift tube under steady-state condition was solved for the case of an immovable virtual cathode (the inertial coordinate system K). To find a solution for a movable VC let us pass on to an inertial system K' which moves rightward along the drift tube with a velocity V. With our configuration of the magnetic field (a uniform leading external magnetic field and an azimuthal self-magnetic field of the electron beam which is uniform along the drift channel), the virtual cathode in the system K' will then move leftward with a velocity V. With the radial component of the magnetic field, there is no point in going to the inertial system K', since the tangential component of the electric field strength therewith appears in the drift tube.

Let there be a tubular beam of radius R_b, electron velocity ν, and current I in the inertial system K (Fig. 1). Then, the electric field strength at the external beam boundary

$$E_r = \frac{2I\gamma}{c\sqrt{\gamma^2 - 1}} \frac{1}{R_b}, \tag{1}$$

and the magnetic field

$$H_\varphi = \frac{2I}{c} \frac{1}{R_b}. \tag{2}$$

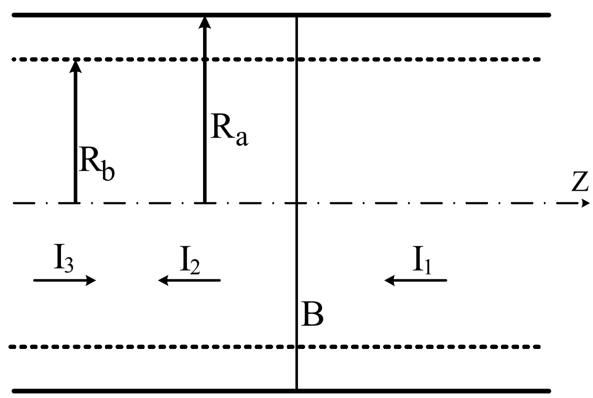

Fig. 1. Schematic of the drift channel with a tubular electron beam: R_a – tube radius, R_b – beam radius; I_1, I_2, I_3 – the current of the beam behind the VC, the injection current, and the current of the beam reflected from the VC (system K, the VC is in the plane B)

In the inertial system K', which moves along the z-axis with an absolute velocity V, the Lorentz transformations for the electric and magnetic filed strengths have the form [6]

$$E'_r = \frac{E_r \pm \frac{V}{c} H_\varphi}{\sqrt{1 - \frac{V^2}{c^2}}} = \gamma\left(E_r \pm \frac{V}{c} H_\varphi\right), \tag{3}$$

$$H'_\varphi = \gamma'\left(H_\varphi \mp \frac{V}{c} E_r\right), \tag{4}$$

where $\gamma' = 1\big/\sqrt{1 - V^2/c^2}$ is a relativistic factor which corresponds to the velocity V of the system K', the upper index stands for the motion of the system K' in the same direction as the beam particles and the lower index for that in opposition to them.

According to (2),

$$I' = \frac{cRH'}{2} = I\gamma'\left(1 \mp \frac{\gamma\sqrt{\gamma'^2 - 1}}{\gamma'\sqrt{\gamma^2 - 1}}\right). \qquad (5)$$

In the system K' we will have the currents

$$I_1' = I_1\gamma\left(1 \mp \frac{\gamma_1\sqrt{\gamma^2 - 1}}{\gamma\sqrt{\gamma_1^2 - 1}}\right),$$

$$I_2' = I_2\gamma\left(1 \mp \frac{\gamma_2\sqrt{\gamma^2 - 1}}{\gamma\sqrt{\gamma_2^2 - 1}}\right), \qquad (6)$$

$$I_3' = I_3\gamma\left(1 \pm \frac{\gamma_2\sqrt{\gamma^2 - 1}}{\gamma\sqrt{\gamma_2^2 - 1}}\right),$$

instead of I_1, I_2, I_3 (Fig. 1). Using the relativistic law of velocity summation, the relativistic factors for the beams in the system K' are easily obtainable

$$\gamma_1' = \gamma\gamma_1 \mp \sqrt{\gamma^2 - 1}\sqrt{\gamma_1^2 - 1},$$

$$\gamma_2' = \gamma\gamma_2 \mp \sqrt{\gamma^2 - 1}\sqrt{\gamma_2^2 - 1}, \qquad (7)$$

$$\gamma_3' = \gamma\gamma_2 \pm \sqrt{\gamma^2 - 1}\sqrt{\gamma_2^2 - 1}.$$

With a knowledge of relativistic factors γ' (7) and electric field strength (1), the applied voltages (Γ') for beams 1, 2, 3 in the system K' are calculated

$$\Gamma_2' = \gamma\gamma_2 \mp \sqrt{\gamma^2 - 1}\sqrt{\gamma_2^2 - 1} + \frac{2\gamma}{I_0}\ln\left(\frac{R_a}{R_b}\right)\left[\frac{(I_2 + I_3)\gamma_2}{\sqrt{\gamma_2^2 - 1}} \pm \frac{(I_2 - I_3)\sqrt{\gamma^2 - 1}}{\gamma}\right],$$

$$\Gamma_1' = \gamma\gamma_1 \mp \sqrt{\gamma^2 - 1}\sqrt{\gamma_1^2 - 1} + \frac{2\gamma I_1}{I_0}\ln\left(\frac{R_a}{R_b}\right)\left[\frac{\gamma_1}{\sqrt{\gamma_1^2 - 1}} \pm \frac{\sqrt{\gamma^2 - 1}}{\gamma}\right], \qquad (8)$$

$$\Gamma_3' = \gamma\gamma_2 \pm \sqrt{\gamma^2 - 1}\sqrt{\gamma_2^2 - 1} + \frac{2\gamma}{I_0}\ln\left(\frac{R_a}{R_b}\right)\left[\frac{(I_2 + I_3)\gamma_2}{\sqrt{\gamma_2^2 - 1}} \pm \frac{(I_2 - I_3)\sqrt{\gamma^2 - 1}}{\gamma}\right].$$

The formulae and the values of currents and relativistic factors obtained for the standard states of an electron beam with an immovable virtual cathode provide a possibility of completely solving the problem with a moving VC. In a formal way, the derived solutions hold true for any VC velocity lower than the velocity of light. However, to each voltage of the drift tube there corresponds its own limiting VC velocity.

3. Results

Let us determine the maximum velocity $V_{\max}^L$ of the VC moving leftward, assuming that this velocity can not higher than that of the electron beam behind the VC. At high values of the VC velocity, the particles change their direction that comes in conflict with the conditions of the initial problem. In the system K the beam which has passed through the VC is on the left branch of the transportation curve [5], and therefore its velocity ranges from zero to a value corresponding to the relativistic factor $\gamma_1 = \Gamma^{1/3}$. Hence, the maximum velocity of the VC moving leftward $V_{\max}^L$ corresponds to $\gamma_1 = \Gamma^{1/3}$, i.e., to the case where the beam passes through the VC without reflection. In this limiting case, the beam in the system K' stops. In so doing, $\Gamma_2' = \Gamma^{2/3}$. But Γ_2' is the relativistic factor which corresponds to the applied voltage in the system K'. Consequently, $\gamma_{\max}^L = \Gamma^{1/3} = \sqrt{\Gamma_2'}$ and

$$\gamma_{\max}^L = c\frac{\sqrt{\Gamma_2' - 1}}{\sqrt{\Gamma_2'}} = c\sqrt{1 - \frac{1}{\Gamma_2'}}. \qquad (9)$$

For the motion of the VC rightward, we will assume that the reflected beam travels to the left and at $V_{\max}^R$ it stops. Then, from (7) we have $\gamma_{\max}^R = \gamma_{2\max}$, because in the system K the injected and reflected beams have equal values of γ. ever, $\gamma_{2\max} = \gamma_F$, and from (8) we obtain

$$\Gamma_2' = \gamma_F^2 + \Gamma\gamma_F - 1, \qquad (10)$$

where $\gamma_F = \sqrt{2\Gamma + 1/4} - 1/2$.

These relations determine the limiting velocity of the VC moving rightward $\gamma_{\max}^R = f(\Gamma_2')$. Unfortunately, this velocity has to be calculated only numerically (Fig. 2).

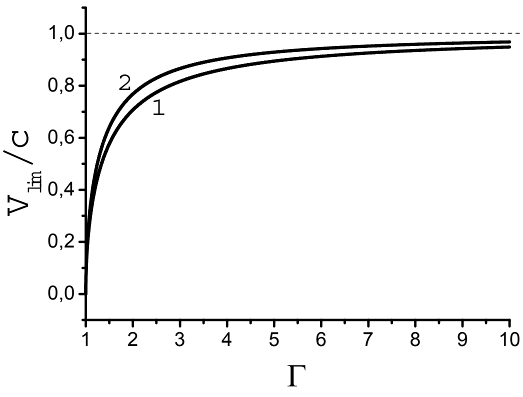

Fig. 2. Limiting velocities of the VC moving leftward (1) and rightward (2) versus the drift tube voltage Γ

In the general case, the VC velocity γ depends on many parameters. Fig. 2 shows γ versus the injection current I_2' at a constant current I_1' of the beam behind the VC for $\Gamma_2' = 2$, $R_a =$ $R_b =$,. This dependence has been calculated by formulae (6–8). It can be seen from this figure that at the specified current I_1' the dependence $\gamma(I_2')$ has two branches: a "slow" branch with low VC velocities at $I_{Tr} < I_2' < I_{2\max}'$ and a "rapid" one which is described by the upper branch at $I_{2\max}' < I_2' < I_1'$. For high injection currents ($I_{Tr} < I_2' < I_{2\max}'$), both solutions are possible, whereas for low injection currents ($I_1' < I_2' < I_{Tr}$) only those described by the "rapid" branch. At injection currents higher than the maximum one $I_2' > I_{2\max}'$, no steady-state of the beam with a constant VC velocity is revealed.

Analysis has shown that the conditions appropriate to the "slow" branch can be realized if the magnetized beam is injected to a narrow section of the two-stage tube from one side. As demonstrated in [3], in this case for every value of the current behind the VC there is a critical value of the injection current I_{Tr} (the transition current), at which the VC starts to move to the plane of the beam injection. The values of these currents correspond to the points of intersection of the curves, which describe the "slow" branch, and the I_2' axis. For instance, for $I_1' = 0$ the curve intersects the x-axis at the point $I_2' = I_{Tr} = I_F/2$ that corresponds to the minimum value of the transition current and at its maximum values $I_1' = I_{1\lim}$ this dependence degenerates into a point.

Note that the point of intersection of this curve with the z-axis corresponds to the limiting velocity of the VC moving leftward to the injection region $\gamma_{\lim} = 1\Big/\sqrt{1-(V_{\lim}^L)^2/c^2}$, where $V_{\lim}^L$ is determined by formula (9) that supports the above assumption that the limiting value of the VC velocity is equal to the electron velocity of the beam behind the VC.

Figure 3 shows the values of VC velocities determined in simulating the electron beam injection into a two-stage tube by the KAPAT code [7]. It can be seen in this figure that these values (even to the point of the VC velocities γ_m which correspond to the maximum injection current $I_2' = I_{2\max}'$) find a rather good agreement with the results of calculations by the formulae derived in this work.

In this work we have not investigated the conditions and patterns of electron beam injection which allows practical realization of the modes of the VC motion described by the "rapid" branch of its I_2' dependence. This does not, however, exclude the possibility of realizing them, e.g., with the use of many-sided injection patterns or patterns of electron injection into a drift channel through the lateral surface.

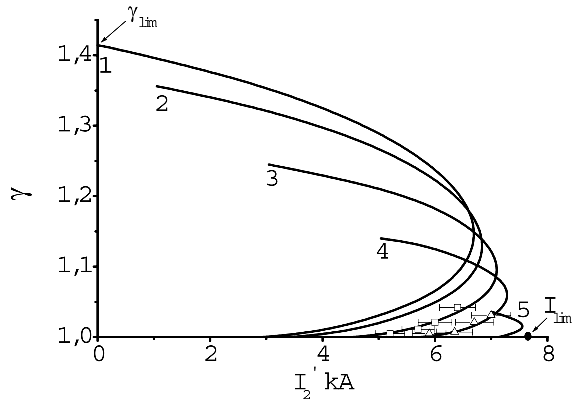

Fig. 3. VC velocities versus the current ratio for $\Gamma = 2$; $R_1 = 1$ cm; $R_b = 0.61$ cm. $I_1' = 0$, 1, 3, 5, 7 kA – curve 1, 2, 3, 4, 5, respectively. Dots indicate the results of calculations by the PIC code KARAT: □ – for the beam current behind the VC $I_1' = 3$ kA, Δ – for $I_1' = 5$ kA.

$$\gamma_{\lim} = 1\Big/\sqrt{1-(V_{\lim}^L)^2/c^2}$$

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