Approximation of the Limiting Self-Magnetic Insulation Current in MITL

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Abstract – In this work the assumption is made that in MITL in the mode of self-magnetic insulation the limiting current occurs at which electrons at the wave front take paths tangential to the anode surface. This makes it possible to use the law of conservation of energy in the system and derive equations for the relativistic factor γ_m which corresponds to the voltage at the external boundary of the electron layer and to determine the main characteristics of MITL in the mode of self-magnetic insulation.

1. Introduction

In the last few decades, high-current electronics has faced the problem with respect to determination of the parameters of a self-magnetic insulation wave (SMIW) in a transmission line (Fig. 1).

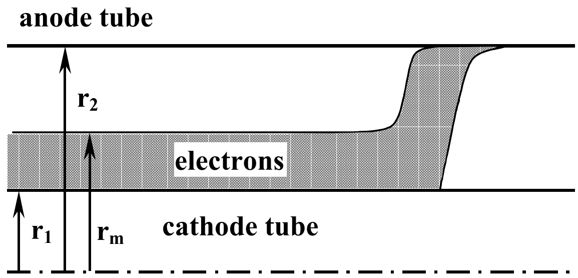

Fig. 1. MITL geometry: r_1, r_2 – cathode and anode radii, r_m – external radius of the electron layer

With a cathode of limitless emissivity, a current-carrying electron layer appears in such a wave. The voltage in the wave U and that in the electron layer U_m or its corresponding relativistic factor $\gamma_m = 1 + eU_m/mc^2$ (*e, m, c* are, respectively, the electric charge and mass of an electron, and the velocity of light in vacuum) are determined by the main MITL characteristics in the regime of self-magnetic insulation: the total current, the cathode current, and the leakage current to the anode.

In the majority of papers devoted to the theory of self-magnetic insulation of MITL, the relativistic factor γ_m is found from an empirical condition of minimum MITL current [1].

In this paper, a proposal is made to determine γ_m by considering the processes occurring at the wave front, which is likely to be a "regulator" of the SMIW parameters. In this way one can derive the equations for γ_m from the law of conservation of energy in the system (or from the law of conservation of the longitudinal momentum component).

2. Theory

Let there be a coaxial line whose internal tube plays the role of a cathode (Fig. 1). Assume that the SMIW parameters are time-invariant and the electron layer behind the wave front is homogeneous such that the solution obtained in the hydrodynamic approximation [2] holds true for it.

Let us take up the processes occurring at the SMIW front in an inertial system K', which moves with a front velocity V_f relative to an immovable laboratory coordinate system. In this system, the SMIW front is assumed to be a stationary one and the electrodes to have no potential difference. Hence, the electrons which leave the cathode with a tangential velocity equal to the wave front velocity V_f will fly to the anode with the same velocity in absolute value.

As the current is increased, the electrons will arrive at the anode at large angles to the normal. In the limiting case, which corresponds to the limiting current $I_{\lim}$ in the MITL (the limiting current approximation $I = I_{\lim}$), all electrons at the SMIW front moves at a tangent to the anode, i.e. in the system K', the velocity of the electrons at the anode is opposite to that at the cathode. With a further increase in current, the electrons get into the regime of self-magnetic insulation and are no longer tangent to the anode (the front disappears).

In this approximation ($I = I_{\lim}$), it is easy to calculate the electron energy at the anode in the laboratory inertial system K, the current and thus the power to the anode in the SMIW. The SMIW power minus the power to the anode equals the energy density per unit length MITL multiplied by the front velocity V_f. This expression relates U to γ_m, and it is the one from which $\gamma_m(U)$ is found in the limiting current approximation.

In the hydrodynamic approximation the electric and magnetic field strengths in the electron layer are determined by the expressions [2]

$$E_r = \frac{H_c r_1}{r} sh(\alpha \ln r / r_1), \tag{1}$$

$$H_\theta = \frac{H_c r_1}{r} ch(\alpha \ln r / r_1), \tag{2}$$

where H_c is the magnetic field strength at the cathode; $\alpha = eH_c r_1 / mc^2$, r_1 is the cathode radius (Fig. 1). The average electron velocity V_b and γ_m therewith are related as

$$V_b = c(\gamma_m - 1)^{1/2} / (\gamma_m + 1)^{1/2}, \quad (3)$$

$$\gamma_m = ch(\alpha \ln r_m / r_1), \quad (4)$$

where r_m is the internal radius of the electron layer (Fig. 1).

The total MITL current I, the electron layer current I_b and the cathode current I_c are determined by the expressions

$$I = \frac{H_c r_1 c}{2}\gamma_m = \frac{I_0 \gamma_m}{2\ln(r_2/r_1)}[\ln(\gamma_m + \sqrt{\gamma_m^2 - 1}) + \frac{\Gamma - \gamma_m}{\sqrt{\gamma_m^2 - 1}}], \quad (5)$$

$$I_b = I(\gamma_m - 1)/\gamma_m, \quad (6)$$

$$I_c = I/\gamma_m, \quad (7)$$

where $I_0 = mc^3/e \approx 17\,\text{kA}, \ \Gamma = 1 + eU/mc^2$.

The relation between γ_m and Γ, as a rule, is found from the condition of the minimum total current in the MITL [1]:

$$dI/d\gamma_m = 0. \quad (8)$$

Taking into account (5), it follows that

$$\ln(\gamma_{m\min} + \sqrt{\gamma_{m\min}^2 - 1}) = (\Gamma - \gamma_{m\min})/(\gamma_{m\min}^2 - 1)^{3/2}. \quad (9)$$

Let us find a relation between γ_m and Γ from the law of conservation of energy. The energy flow (power) in the MITL cross section in a homogeneous region is equal to

$$W = IU = \frac{I_0 H_c r_1}{2}\gamma_m(\Gamma - 1), \quad (10)$$

and the energy density per unit length which includes the density of the kinetic electron energy and the field energy in the electron layer and in the gap separating the latter and the anode

$$\varepsilon = \frac{I_0 H_c r_1}{4c}[2\gamma_m\sqrt{\gamma_m^2 - 1} + \frac{(2\gamma_m^2 - 1)(\Gamma - \gamma_m)}{\sqrt{\gamma_m^2 - 1}} - 2\sqrt{\gamma_m^2 - 1} + \ln(\gamma_m + \sqrt{\gamma_m^2 - 1})]. \quad (11)$$

The SMIW velocity

$$V_f = c\frac{\sqrt{\gamma_m^2 - 1}\,(\Gamma - 1)}{\Gamma\gamma_m - 1} \quad (12)$$

is higher than the average electron velocity in the layer V_b. Therefore, the current I_c is spent in part on charging the electron layer, while the rest part I_L (the leakage current) flows to the anode

$$I_L = I_c - I_b(\frac{V_f}{V_b} - 1) = \frac{H_c r_1 c\,(\Gamma + \gamma_m^2 - \gamma_m - 1)}{2\,(\gamma_m\Gamma - 1)}. \quad (13)$$

In the limiting current approximation and according to the relativistic law of velocity summation, the electrons with a current I_L, arriving at the anode has the velocity

$$V_1 = \frac{2V_f}{(1 + V_f^2/c^2)}, \quad (14)$$

and the relativistic factor corresponding to V_1

$$\gamma_1 = \frac{1}{\sqrt{(1 - V_1^2/c^2)}} = \frac{(\gamma_m^2 - 1)(\Gamma - 1)^2 + (\gamma_m\Gamma - 1)^2}{(\gamma_m\Gamma - 1)^2 - (\gamma_m^2 - 1)(\Gamma - 1)^2}. \quad (15)$$

Consequently, the power carried to the anode by the current I_L (5)

$$W_1 = \frac{I_L}{e}mc^2(\gamma_1 - 1) = \frac{I_0 H_c r_1(\Gamma + \gamma_m^2 - \gamma_m - 1)}{2\,(\gamma_m\Gamma - 1)}(\gamma_1 - 1). \quad (16)$$

From the law of conservation of energy we have

$$W - W_1 = \varepsilon\, V_f \quad (17)$$

and upon cancellation we obtain the relation which determine γ_m in the limiting MITL current approximation ($I = I_{\lim}$):

$$2\,\gamma_{m\lim}(\Gamma - 1) - 2\,(\gamma_1 - 1)\frac{(\Gamma + \gamma_{m\lim}^2 - \gamma_{m\lim} - 1)}{(\gamma_{m\lim}\Gamma - 1)} = \frac{\Gamma - 1}{\gamma_{m\lim}\Gamma - 1}(2(\gamma_{m\lim})^2(\Gamma - 1) - \Gamma - \gamma_{m\lim} + 2 + \sqrt{(\gamma_{m\lim})^2 - 1} \times \ln(\gamma_{m\lim} + \sqrt{(\gamma_{m\lim})^2 - 1})), \quad (18)$$

where γ_1 is found by the formula (15).

3. Results

Figure 2 presents the dependence of $\gamma_{m\,\lim}$ on the wave voltage U calculated by formula (18). For comparison the U-dependence of $\gamma_{m\,\min}$ obtained by formula (9) in the minimum MITL current approximation ($I = I_{\min}$) is also presented in this figure. It can be seen that in the two approximations the values of γ_m and, consequently, those of the voltage in the electron layer U_m differ considerably, in particular, in the range of high wave voltages U. This causes a relative increase in current in the electron layer ("covering") and a several-fold decrease in cathode current.

At the same time, the MITL impedance varies to a lesser degree (Fig. 3): calculations in the limiting current approximation $Z_{\lim}$ reveal that in the range of low voltages ($U < 3$ MV) the impedance decreases by tens of percents (at $U = 5$ MV by 10%) and in that of high values by several percents (at $U = 15$ MV by 5%), as compared to the calculations of $Z_{\min}$ in the minimum current approximation. Therefore, the total MITL current changes, but slightly (Fig. 2).

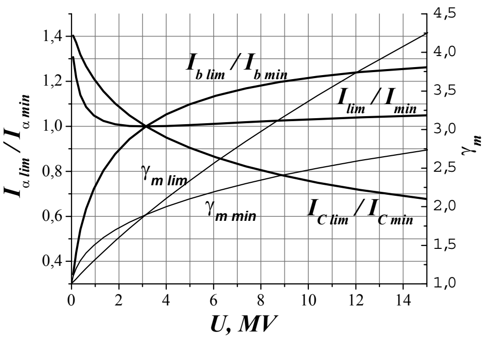

Fig. 2. Relativistic factor γ_m, which corresponds to the potential at the electron layer boundary (thin lines) and the ratio of the MITL currents (thick lines) versus the wave voltage. I, I_C and I_b – total current, cathode tube current, and electron layer current. The index lim stands for the limiting current approximation and the index min for the minimum current approximation

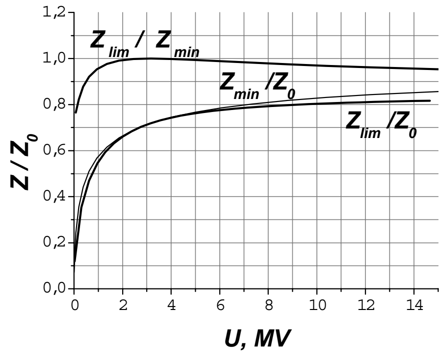

Fig. 3. MITL impedance calculated in the limiting $Z_{\lim}$ / Z_0 (thick lines) and minimum $Z_{\min}$ / Z_0 (thin lines) current approximations and their ratio $Z_{\lim}$ / $Z_{\min}$ versus the voltage in the wave U. Z_0 – "cold" line impedance

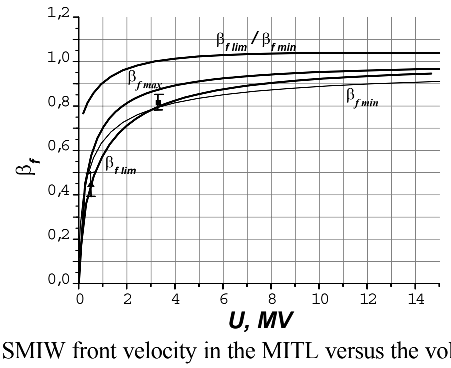

Fig. 4. SMIW front velocity in the MITL versus the voltage U. $\beta_{f\,\lim}$ and $\beta_{f\,\min}$ – calculation in the limiting and minimum current approximations, respectively. $\beta_{f\max} = [(\Gamma - 1)/(\Gamma+1)]^{1/2}$ – limiting wave velocity corresponding to $\gamma_m = \Gamma$. Experiment for $U = 0.46$ MV [3]; $U = 3.4$ MV [4]

Figure 4 shows the dependence of the SMIW front velocity $\beta_f = V_f/c$ on the wave voltage which has been calculated by formula (12) in two approximations in question. The greatest (up to 20%) difference in the front velocities is observed in the range of low voltages where the limiting current approximation predicts much lower velocities. For comparison the same figure shows the voltage dependence of the limiting front velocity described by formula (12) at $\gamma_m = \Gamma$: $\beta_{f\max} = (\Gamma - 1)^{1/2}/(\Gamma + 1)^{1/2}$ and also the results of two experiments where β_f has been measured for the voltages $U = 0.46$ MV [3] and $U = 3.4$ MV [4]. These results were taken from [5]. The both calculated curves agree well with data on measuring the velocity for $U = 3.4$ MV near which the curves $\gamma_{m\,\lim}(U)$ and $\gamma_{m\,\min}(U)$ intersect (Fig. 2). However, in the range of low voltages the experimental values of the velocity show a better agreement with calculations by formulae (12), (18). The values of γ_1 have been calculated by formula (15). It can be seen from this figure that for voltages which are normally used in experiments ($U < 15$ MV) the electrons at the wave front acquire less than 60% of the field energy that agrees qualitatively with experiments.

The kinetic energy of the electrons arriving with the wave front at the anode on the potential energy at the cathode, $\alpha = (\gamma_1 - 1)/(\Gamma - 1)$ is shown in Fig. 5.

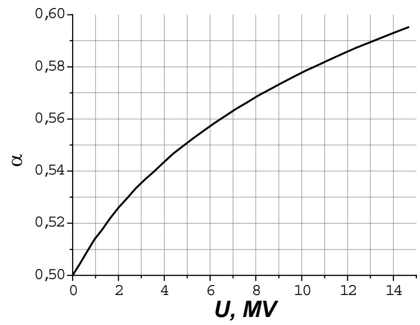

Fig. 5. Ratio of the kinetic energy of the electrons at the SMIW front arriving at the anode to eU versus U

Comparison of the results obtained in the limiting current approximation with those of the theory based on the minimum MITL current approximation has shown that the both approaches provide close estimates of the total MITL current (Fig. 2). However, the values of γ_m differ considerably that can be used for experimental test of the theory.

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