# Formation of the Cylindrical Plasma Flow with Enhanced Ion Charge States in a Vacuum Arc in an Axial Magnetic Field[1]

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**Abstract – Supersonic vacuum arc plasma motion in the presence of a strong axial magnetic field has been studied theoretically. It is shown that a quasi-cylindrical plasma jet with increasing electron temperature and ion charge state occurs until plasma slows down to subsonic velocity. Limiting electron temperature corresponding to this critical point is controlled by the initial ion energy and equal to a triple value of electron temperature in the cathode spot region.**

## 1. Introduction

As it is known the application of an axial magnetic field to the vacuum arc discharge leads to significant enhancement of ion charge state even at the low currents [1]. It may be thought that an increase in electron temperature is the main reason of a rise in ionicity. Indeed the model calculations have predicted the enhancement of electron temperature in the inter-electrode gap plasma in the presence of the external magnetic field [2, 3]. Recently such enhancement has been discovered experimentally [4]. On the other hand it was observed that the plasma flow transforms to a cylindrical channel that diameter is independent of the distance to the cathode [1, 5] if a strong axial magnetic field is applied. Unabated growth of the electron temperature should take place along a cylindrical plasma jet. But the possibility of the occurrence of such a jet is not studied in details. This question is beyond the scope of the models [2, 6] and has been considered in [3] using a rough approximation of unchanged longitudinal plasma velocity. Therefore the purpose of present paper is to clarify conditions of cylindrical plasma jet formation and to estimate a maximum value of electron temperature that may be achieved in the vacuum arc operating in a uniform axial magnetic field. A long arc discharge with the inter-electrode distance essentially prevailing over the cathode diameter is studied. Model developed is mostly adequate to vacuum arc with an annular anode.

## 2. Basic Equations

We begin by writing hydrodynamic equations for a plasma jet ignoring the viscosity and thermal conductivity. In cylindrical coordinates $r$, $\varphi$, $z$ the continuity and motion equations have the following form:

$$\frac{1}{r}\frac{\partial}{\partial r}(r\rho V_r) + \frac{\partial}{\partial z}(\rho V_z) = 0; \quad (1)$$

$$\rho\left(V_r\frac{\partial V_r}{\partial r} + V_z\frac{\partial V_r}{\partial z}\right) = -\frac{\partial P}{\partial r} + \frac{j_\varphi B_z}{c} - \frac{j_z B_\varphi}{c}; \quad (2)$$

$$\rho\left(V_r\frac{\partial V_z}{\partial r} + V_z\frac{\partial V_z}{\partial z}\right) = -\frac{\partial P}{\partial z} + \frac{j_r B_\varphi}{c}, \quad (3)$$

where $\rho$ and $P$ are the plasma density and pressure; $V_r$ and $V_z$ are radial and axial components of the ion velocity $\mathbf{V}$; $j_r$, $j_\varphi$, $j_z$ are the radial, azimuthal, and axial components of the current density $\mathbf{j}$; $B_z$ = const is the external magnetic field; $B_\varphi$ is the self-magnetic field generated by the arc current; $c$ is the velocity of light. The energy balance equation for the overall plasma can be written as follows:

$$\nabla\cdot\left(\tfrac{5}{2}P\mathbf{V} + \tfrac{1}{2}\rho V^2\mathbf{V}\right) = \mathbf{j}\cdot\mathbf{E} + \nabla\cdot\left(\tfrac{5}{2}T_e\mathbf{j}/e\right), \quad (4)$$

where $\mathbf{E}$ is the electric field; $T_e$ is the electron temperature (in energy units); and $e$ is the electron charge. Also we use the Ohm's low

$$\mathbf{j} = \sigma\left(\mathbf{E} + \frac{1}{c}(\mathbf{V}\times\mathbf{B}) + \frac{1}{eN_e}\nabla P_e - \frac{1}{ceN_e}(\mathbf{j}\times\mathbf{B})\right) \quad (5)$$

and the charge conservation equation:

$$\frac{1}{r}\frac{\partial}{\partial r}(r\,j_r) + \frac{\partial}{\partial z}(j_z) = 0, \quad (6)$$

where $P_e = N_e T_e$ and $N_e$ are the electron pressure and density; $\sigma = e^2 N_e \tau_e / m_e$ is the plasma conductivity; $m_e$ is the electron mass, $\tau_e = \frac{3}{4}(m_e(kT_e)^3/2\pi)^{1/2}/(e^4 Z_i^2 N_i \ln\Lambda)$ is the electron-ion collision time; $N_i = N_e / Z_i$ is the ion density; $Z_i$ is the mean ion charge; $\ln\Lambda$ is the Coulomb logarithm. The self-magnetic field is determined by equation

$$B_\varphi = \frac{4\pi}{cr}\int_0^r j_z r\,dr. \quad (7)$$

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### 3. Method of Solution

To simplify the problem, the following main assumptions are used.

(1) The current channel and the plasma flow are the same and have the equal cross-section $S = \pi R^2$, where $R(z)$ is their radius varying with a distance $z$ from cathode surface. Thus we have $\mathbf{j} \times \mathbf{V} = 0$.

(2) The particle density, pressure and temperature and axial component of ion velocity and current density are taken to be uniform over the flow cross-section and dropping to zero in a narrow region near plasma jet boundary. For example we have $\rho(r,z) = \rho(z)\,\theta(Rr)$, where θ is the theta-function.

(3) Ion pressure is neglected in comparison with electron pressure due to the low value of ion temperature. Thus $P = P_e$ holds.

(4) The anode is considered to be a passive collector of the charge and mass fluxes.

Using (1) we can express the radial component of ion velocity through its side boundary value as follows:

$$V_r = V_R\, r/R\,,\; V_R = V_z dR/dz. \tag{8}$$

Under assumption made, we can find from (6)

$$j_r = j_R r/R,\; j_R(z) = j_z dR/dz; \tag{9}$$

$$j_z = -I/\pi R^2. \tag{10}$$

Using Ohm's low (5) we also obtain

$$j_\varphi = \omega_e \tau_e (j_r e N_e V_r), \tag{11}$$

where $\omega_e = eB_z/(m_e c)$ holds.

As it is seen from (9)–(11) all the components of the current density are expressed in terms of an arc current $I$ and a jet radius $R$. From (7) we also find

$$B_\varphi = B_R r/R\,,\; B_R = 2I/(cR). \tag{12}$$

Multiplying (1)–(3) by $dS = 2\pi r dr$, integrating from 0 to ∞, and using (8)–(11) we can transform the mass and momentum conservation equations to the one-dimensional form:

$$\frac{d}{dz}(\rho V_z S) = 0; \tag{13}$$

$$\rho V_z S\,\frac{dV_R}{dz} = \frac{3PS}{R} - \frac{2}{R}\left(\frac{I}{c}\right)^2 - \omega_e \tau_e I B_z \frac{dR}{dz}; \tag{14}$$

$$\rho V_z S\,\frac{dV_z}{dz} = -\frac{d(PS)}{dz} + \left(\frac{I}{c}\right)^2 \frac{d\ln R}{dz}. \tag{15}$$

We can determine the electric field **E** from the Ohm's law (5) and substitute it to the energy balance (4). Integrating we obtain

$$\frac{d}{dz}\left((\tfrac{5}{2}P + \tfrac{1}{2}\rho(V_z^2 + \tfrac{1}{2}V_R^2))V_z S\right) = \frac{I^2}{\sigma S}\left(1 + \frac{j_R^2}{2j_z^2}\right) - \frac{5I}{2e}\frac{dT_e}{dz} + \frac{I}{eN_e}\frac{dP}{dz}. \tag{16}$$

Equations (13) and (15) have the evident solutions:

$$\rho V_z S = G = \text{const}; \tag{17}$$

$$GV_z + PS - (I/c)^2 \ln R = \text{const}. \tag{18}$$

Last one is identical to the Bernoulli equation for a steady-state flow. Using the erosion coefficient $\delta = eG/m_i I = \text{const}$ and taking into account the equalities $N_e = Z_i N_i$ and $\rho = m_i N_i$ with $m_i$ as the ion mass we can define the electron density as follows:

$$N_e = Z_i I\delta/(eV_z S). \tag{19}$$

The mean ion charge state $Z_i$ can be calculated using the ion species balance equations [7].

### 4. Boundary Conditions

In the vacuum arc the plasma is generated in the cathode micro-spots spread over cathode surface. At the distance of order of 100 μm from cathode the separate plasma micro-jets merge into a single jet (a plasma flow) that is the subject of present investigation. Thus we specify the boundary conditions in this near cathode region assuming the initial flow cross-section $S_0 = \pi R_0^2$ to be equal to the area of cathode spot. We use, as input data, the current value $I$, the ion erosion coefficient $\eta_0 = Z_{i0}\delta$, the charge and velocity of ions $Z_i = Z_{i0}$ and $V_z = V_0$ that may be taken from measurements in a low current arc. We also define the radial velocity as $V_R = V_0 \tan\alpha$ with jet angle α = 0–45°. The initial electron temperature may be estimated as $T_{e0} \approx T_{\text{spot}}/2$ [8], where $T_{\text{spot}}$ is the electron temperature of the cathode spot plasma [9]. The electron and ion densities at $z = 0$ may be found from (19).

It is important to note that the initial ion velocity depends on the electron temperature of a near cathode plasma according to equation

$$V_0 \approx 3.5(5Z_{i0}T_{\text{spot}}/3m_i)^{1/2} \tag{20}$$

obtained theoretically [8] and supported by the measurements [10]. Hence the Mach number appears to be approximately equal to $M_0 = V_0/C_S \approx 6.4$ for a root part of the plasma flow regardless of the cathode material. Here $C_S = (Z_{i0}T_{e0}/m_i)^{1/2}$ is the sound velocity. Thus we have the problem of supersonic plasma expansion into vacuum ambient in the presence of the axial magnetic field.

Below we shall consider the plasma jet carrying 100 A current by the following reasons. (i) Up to this current value only one cathode spot of well-known radius $R_0 \approx 0.2$ mm [11] is present. (ii) When the current is increased beyond 100 A, the cathode spot splits and several independent arc channel are formed [5]. For Ti-arc considered plasma parameters are $\eta_0 = 0.085$, $Z_{i0} = 2$, $V_0 = 1.5\cdot10^4$ m/s, $T_{spot} = 3.2$ eV.

## 5. Calculation Results

Equations (14), (16)–(19) have been solved numerically for the axial magnetic field ranging from zero to 2000 G. Plasma parameters calculated are plotted versus the distance from cathode in Fig. 1.

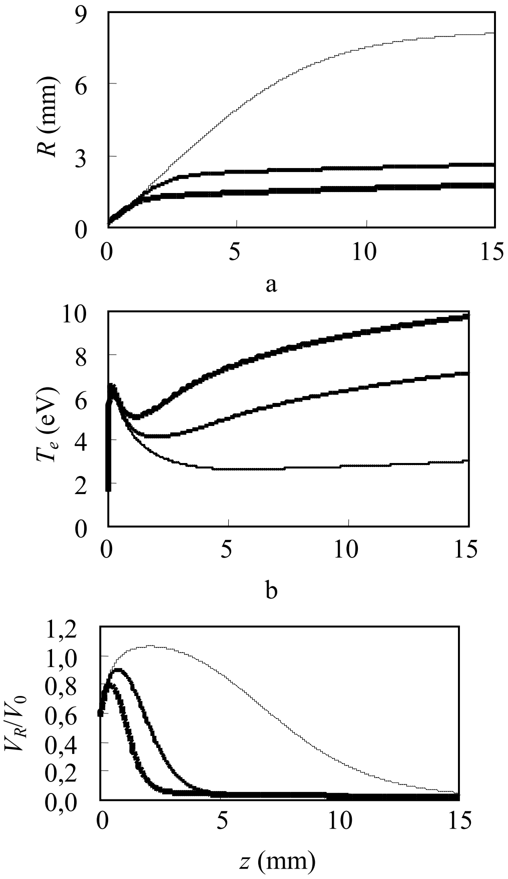


Fig. 1. Plasma jet radius (a), electron temperature (b), and radial plasma velocity (c) as a function of distance from cathode for Ti-arc at $B_z = 1000$ G ( ▬ ), $B_z = 500$ G ( — ), and $B_z = 100$ G ( — )

It is seen that application of an axial magnetic field leads to decrease of the radial velocity (Fig. 1c) and retards a radial plasma expansion. As a result the plasma jet of a shape close to cylindrical one is formed (Fig. 1a). Characteristic feature of this case is the electron temperature increase along a jet (Fig. 1b). With enhancement of the field strength $B_z$ the onset of cylinder formation moves toward the cathode but the rate of plasma constriction decreases. Therefore further gain in field strength does not lead to further significant radius decrease and electron temperature increase after achievement of some limiting value of $B_z$. This conclusion has been made earlier on the base of a more simple model [3]. Recent measurements of electron temperature [4] support these model results.

On Fig. 2 the dependence of mean ion charge state on magnetic field strength is presented. Calculations have been fulfilled for the initial jet angle varying from zero to $\alpha = 45°$. Values of $Z_i$ obtained are found to be not sensitive to the angle value. As it is seen from Fig. 2 the measurements in a low-current vacuum arc [1] agree closely with the calculations. When Fig. 2 is compared with Fig. 1b it is apparent that the ion charge enhancement is caused by the rise in electron temperature with an increase in magnetic field.

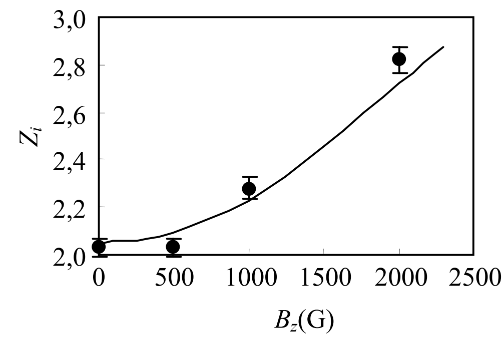


Fig. 2. Growth of mean ion charge with axial magnetic field for Ti-arc: solid line represents the calculation results, circles corresponds to the measurements [1]

## 6. Analysis of the Plasma Motion Equations

To analyze the plasma jet behavior in a strong magnetic field it is convenient to use the non-dimensional variables $x = z / R_0$, $a = R / R_0$, $v = V_z / V_0$, $w = V_R / V_0$, $n = N_e / N_{ei0}$, $t = T_e / T_{e0}$, and $q = Z_i / Z_{i0}$. Then we obtain from (8), (14), (17), (18)

$$nva^2 = 1\,; \qquad (21)$$

$$v = 1 + \gamma_0(1 - qnta^2) + \gamma_1 \ln a\,; \qquad (22)$$

$$\frac{dw}{dx} = 3\gamma_0 qnta - 2\gamma_1 / a - \gamma_2 a^2 t^{3/2} w / q^2, \qquad (23)$$

where $\gamma_0 = 1/M_0^2 = C_S^2/V_0^2$, $\gamma_1 = C_A^2/V_0^2$, $\gamma_2 = \varpi_e \tau_{e0} \omega_i \tau_{i0} / \delta$ are defined. Here $C_A = B_{R0}/(4\pi\rho_0)^{1/2}$ and $\tau_{e0}$ are the Alfven velocity and the collision time at the root cross-section of jet; $B_{R0} = 2I/(cR_0)$; $\omega_i = eB_z/(m_i c)$; $\tau_{i0} = R_0/V_0$. As it was mentioned in Section 4, $(V_0/C_S)^2 \approx 40$ holds and we always have a small parameter $\gamma_0 \approx 0.025$. For our case of Ti-arc at

100 A we have $\gamma_1 \approx 0.028$ and $\gamma_2 \approx 2.4 \cdot 10^{-9} B_z^2$, where $B_z$ is taken in G.

If all the parameters $\gamma_0$, $\gamma_1$, $\gamma_2$ are essentially less than unity a conical plasma expansion with $w \approx \text{const}$ takes place at the initial part of the jet. Following growth of the jet radius leads to a rapid increase of the last term in (23) and consequently to decrease of value of $w$. It means that radial plasma expansion is suppressed by the axial magnetic field resulting formation of a cylindrical flow along a field.

As it is seen from (15) or (22) the change of the longitudinal plasma velocity originates from pressure gradient force and Ampere's force. Last one accelerates the plasma in a case of flow cross-section increase ($dR/dz > 0$) and decelerates it in a case of decreasing cross-section. For a cylindrical flow ($dR/dz \approx 0$) Ampere's force action is negligible and the pressure gradient force directed against plasma motion slows the plasma jet down.

## 7. Crisis of Plasma Flow and Limiting Electron Temperature Value

In a case of cylindrical jet the continuous growth of the electron temperature inevitably leads to a problem of transition from supersonic flow to subsonic one. In a process of investigation it has been found that solution of a set of MHD equations used is possible only for a limiting length $L_{\text{cr}}$ of plasma jet. Indeed, solving equations (21), (22), we can obtain

$$v = A + (A^2 - \gamma_0 q t)^{1/2}, \qquad (24)$$

where $2A = 1 + \gamma_0 + \gamma_1 \ln a$ holds. Thus we can see that solution is possible only in the range of temperature values $t \le t_{\text{cr}} = A^2 / \gamma_0 q$ realized at the jet length $L \le L_{\text{cr}}$. For a long jet at the distance $z = L_{\text{cr}}$ crisis of flow occurs. It may be overcome by means of plasma jet transition to a non-stationary mode with oscillating plasma parameters. The electron temperature would be expected to be close to the limiting value $T_{\text{cr}} = T_{e0} t_{\text{cr}}$ achieved in a vacuum arc located in the axial magnetic field. To estimate this value let us suppose $2A \approx 1$ and $q \approx 1$. Then we obtain $T_{\text{cr}} \approx 1/4\gamma_0 = m_i V_0^2 / 4Z_{i0}$. So it is seen that the initial energy and charge of ions control the limiting value of the electron temperature. Because these values are connected by equation (20) with the electron temperature in the cathode spot we can find $T_{\text{cr}} \approx 3T_{\text{spot}}$. To check this simple relationship the maximal values of electron temperature measured at distance $z = 15$ mm and $B_z = 300–600$ G [4, 12] are plotted in Fig. 3 against the electron temperature of the cathode spot plasma [9]. The good agreement with model estimation is evident.

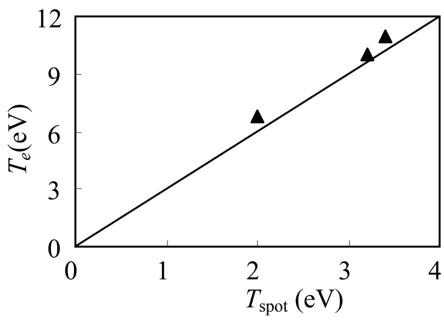


Fig. 3. Dependence of the electron temperature of inter-electrode plasma on the electron temperature in cathode spot. Triangles correspond to measurements of $T_{\text{spot}}$ from [9] and $T_e$ from [4, 12] for C, Ti and U-arc. Solid line represents the dependence $T_e = 3T_{\text{spot}}$

## 8. Conclusion

The consideration presented above shows that in the presence of a strong magnetic field the plasma jet takes a form close to cylindrical one due to suppression of the radial plasma expansion by an axial magnetic field. The enhancement of electron temperature along the axis is a characteristic feature of a cylindrical flow. It is found that continuous growth of the electron temperature inevitably leads to crisis of flow due to plasma jet decelerating. Thus a steady-state plasma expansion may occur only for a jet of limiting length. It is shown that the limiting value of electron temperature is controlled by the initial energy of ions and may be presented as a function of the electron temperature in a cathode spot.

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